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Compensation of input delay that depends on delayed input☆

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ABSTRACT

For nonlinear systems, we develop a PDE-based predictor-feedback control design, which compensates actuator dynamics, governed by a transport PDE with outlet boundary-value-dependent propagation velocity. Global asymptotic stability under the predictor-feedback control law is established assuming spatially uniform strictly positive transport velocity. The stability proof is based on a Lyapunov-like argument and employs an infinite-dimensional backstepping transformation that is introduced. An equivalent representation of the transport PDE/nonlinear ODE cascade via a nonlinear system with an input delay that is defined implicitly through an integral of the past input is also provided and the equivalent predictor-feedback control design for the delay system is presented. The validity of the proposed controller is illustrated applying a predictor-feedback "bang–bang" boundary control law to a PDE model of a production system with a queue. Consistent simulation results are provided that support the theoretical developments.

1. Introduction

Cascades of partial and ordinary differential equations are widely used for modeling of complex dynamics in various engineering applications, such as screw extrusion processes in 3D printing (Diagne & Krstic, 2015), metal cutting processes (Otto & Radons, 2013), moisture in convective flows (Bresch-Pietri & Coulon, 2015), populations (Smith, 1993), transport phenomena in gasoline engines (Bresch-Pietri, Chauvin, & Petit, 2014; Detwiler & Wang, 2006; Guzzella & Onder, 2009; Jankovic & Magner, 2011; Kahveci & Jankovic, 2010), crushing-mills (Richard, 2003), production of commercial fuels by blending (Chebre, Creff, & Petit, 2010), and of stick–slip instabilities during oil drilling (Bekiaris-Liberis & Krstic, 2014; Cai & Krstic, 2015, 2016; Krstic, 2009), to name only a few. Depending on the application, the PDE state may evolve on a time-varying domain (Cai & Krstic, 2015, 2016; Diagne & Krstic, 2015; Diagne, Shang, & Wang, 2016a,b) or its transport coefficient may vary with time (Bresch-Pietri et al., 2014; Otto & Radons, 2013).

The nonlinear predictor-feedback concept, which enables one to design efficient feedback laws that compensate constant input delays arising in nonlinear systems was originally introduced in Krstic (2010a, c) where the PDE backstepping methodology combined with a Lyapunov analysis was exploited to establish stability results. For nonlinear systems with time-varying and state-dependent delays, the analogous control design methodology was developed in Bekiaris-Liberis, Jankovic, and Krstic (2012) and Bekiaris-Liberis and Krstic (2012, 2013a, b), . For linear systems Karafyllis, Malisoff, de Queiroz, Krstic and Yang (2015) and Mazenc and Malisoff (2015) proposed alternative prediction-based approaches. Later, the method was extended to deal with the stabilization problem of nonlinear systems with actuator dynamics governed by a wave PDE with moving boundary that depends on the ODE state (Cai & Krstic, 2015, 2016).

However, the problem of design of predictor-feedback controllers for compensation of input delays that depend on the control input itself is left out in most of the existing contributions. As described in Richard (2003), the design of delay-compensating control laws for such systems of transport PDE/ODE cascades with input-dependent transport coefficient (that appear for example when describing the dynamics of crushing-mill processes (Richard, 2003), recycling CSTR (Albertos & Garcia, 2012), and single-phase marine cooling systems (Hansen, Stoustrup, & Bendtsen, 2013)) remains an open problem. To our knowledge, the result in Bresch-Pietri et al. (2014), which is motivated by the dynamical model of fuel to air ratio (FAR) in gasoline engines, is perhaps the only contribution that covers this particular subject on delay compensation. Due to the dependency of the prediction horizon on the future input values a design that completely compensate the input delay does not seem possible.

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The present work deals with the problem of compensation of transport PDE actuator dynamics with boundary-value-dependent propagation speed in nonlinear systems. Equivalently, the nonlinear ODE's actuator dynamics are described as a *delayed-input-dependent input delay*. Here, the delay function is implicitly given by an integral equation, similarly to Bresch-Pietri et al. (2014), but is dependent on the delayed rather than the current input.

The predictor-feedback control law for both the PDE and the delay system representations of the PDE–ODE cascade system is developed. Our contribution stands as the first one in which actual *compensation* of a delayed-input-dependent input delay is achieved. A global stability result of the closed-loop system is established. The designed compensator is employed for control of PDE models of production systems with a finite buffer size at the end of the production chain (Borsche, Colombo, & Garavello, 2010; Herty, Klar, & Piccoli, 2007; Sun & Dong, 2008).

The paper is organized as follows. In Section 2 the general problem is described and the main result together with a global stability proof, based on a PDE representation of the predictor-feedback control law, is presented in Section 3. An alternative representation of the actuator dynamics as an implicitly defined delayed-input-dependent input delay and the associated delay compensator are given in Section 4. Section 5 is dedicated to the application of the designed control law to a PDE model of production systems enabling a delay-compensating "bang–bang" feedback law. Concluding remarks are stated in Section 7.

2. Problem statement and controller design

We consider the transport PDE/nonlinear ODE cascade system with boundary-value-dependent propagation speed defined as

$$\dot{X}(t) = f\left(X(t), u(0,t)\right), \tag{1}$$

$$\partial_t u(x,t) = v\left(u(0,t)\right) \partial_x u(x,t), \quad x \in (0,D), \tag{2}$$

$$u(D,t) = U(t). \tag{3}$$

where $X \in \mathbb{R}^n, f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is continuously differentiable with $f(0,0) = 0$, and $v : \mathbb{R} \to \mathbb{R}_+$ is continuously differentiable with respect to its argument. Eq. (2) represents the actuation path for the plant (1), located at the boundary $x = 0$, with an actuation device acting at the boundary $x = D$. The initial condition along the actuation path (2) is defined as

$$u(x,0) = u_0(x). \tag{4}$$

We design the following predictor-feedback controller for system (1)–(3)

$$U(t) = \kappa(p(D,t)), \tag{5}$$

$$p(x,t) = X(t) + \int_0^x \frac{1}{v(u(y,t))} f(p(y,t), u(y,t))dy. \tag{6}$$

The implementation of the control law (5), (6) requires measurements of the PDE state $u(x,t)$, $x \in [0,D]$. We emphasize that in the recent papers (Karafyllis, 2011; Karafyllis & Krstic, 2014), the implementation issue of predictor feedback is discussed in detail and various numerical schemes are developed for computation of predictor feedback laws.

For the system (1)–(3), we state the following assumptions:

Assumption 1. The delayed input-dependent propagation speed $v : \mathbb{R} \to \mathbb{R}_+$ is continuously differentiable and there exists a positive constant ε such that

$$v(\alpha) \geq \varepsilon, \quad \text{for all } \alpha \in \mathbb{R}. \tag{7}$$

Assumption 2. There exist a smooth positive definite function Θ and class $\mathcal{K}_\infty$ functions $\mathscr{K}_1$, $\mathscr{K}_2$, and $\mathscr{K}_3$ such that for the plant $\dot{X} = f(X,\omega)$ such that $f(0,0) = 0$, the following hold

$$\mathscr{K}_1(|X|) \leq \Theta(X) \leq \mathscr{K}_2(|X|) \tag{8}$$

$$\frac{\partial \Theta(X)}{\partial X} f(X,\omega) \leq \Theta(X) + \mathscr{K}_3(|\omega|), \tag{9}$$

for all $(X, \omega)^T \in \mathbb{R}^{n+1}$.

Assumption 2 guarantees that system $\dot{X} = f(X,\omega)$ is strongly forward complete with respect to ω.

Assumption 3. System $\dot{X} = f(X, \kappa(X)+\omega)$ is input-to-state stable (ISS) with respect to ω. Moreover, the feedback law $\kappa : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable with $\kappa(0) = 0$.

The definitions of strong forward completeness and input-to-state stability are those from Krstic (2010b) and Sontag (1995), respectively.

3. Main result and stability proof

Theorem 1. *Consider system (1)–(3) together with the control law (5), (6). Under Assumption 1–3, there exists a class $\mathcal{KL}$ function $\mathscr{L}_0$ such that for all initial conditions for which $u_0(x)$ is locally Lipschitz on $[0,D]$ and which satisfy the compatibility condition $u_0(D) = \kappa\left(p(D,0)\right)$, there exists a unique solution to the closed-loop system with $X(t) \in C^1[0,\infty)$ and $u(x,t)$ locally Lipschitz on $[0,D] \times [0,\infty)$, and the following holds for all $t \geq 0$*

$$|X(t)| + \sup_{x\in[0,D]} |u(x,t)| \leq \mathscr{L}_0\left(|X(0)| + \sup_{x\in[0,D]} |u_0(x)|, t\right). \tag{10}$$

The proof of Theorem 1 is established with the help of the following lemmas.

Lemma 1. *The infinite-dimensional backstepping transformation*

$$w(x,t) = u(x,t) - \kappa\left(p(x,t)\right), \tag{11}$$

where $p(x,t)$ is defined in (6), combined with the control law defined in (5), (6), maps the system (1), (2) with the boundary condition (3) into the following target system

$$\dot{X}(t) = f\left(X(t), \kappa\left(X(t)\right) + w(0,t)\right), \tag{12}$$

$$\partial_t w(x,t) = v(w(0,t) + \kappa\left(X(t)\right))\partial_x w(x,t), \quad x \in [0,D] \tag{13}$$

$$w(D,t) = 0. \tag{14}$$

Proof. Differentiation of (6) with respect to t gives

$$\begin{aligned}\partial_t p(x,t) = {} & f(p(0,t), u(0,t)) - \int_0^x f(p(y,t), u(y,t)) \\ & \times \left(\frac{v'(u(y,t))}{v^2(u(y,t))}\partial_t u(y,t)\right) dy \\ & + \int_0^x \frac{1}{v(u(y,t))}\partial_p f(p(y,t), u(y,t))\partial_t p(y,t)dy \\ & + \int_0^x \frac{1}{v(u(y,t))}\partial_u f(p(y,t), u(y,t))\partial_t u(y,t)dy.\end{aligned} \tag{15}$$

Differentiation of (6) with respect to x leads to the following relation

$$\begin{aligned}\partial_x p(x,t) = {} & -\int_0^x f(p(y,t), u(y,t)) \left(\frac{v'(u(y,t))}{v^2(u(y,t))}\partial_y u(y,t)\right) dy \\ & + \int_0^x \frac{1}{v(u(y,t))}\partial_p f(p(y,t), u(y,t))\partial_y p(y,t)dy\end{aligned}$$

$$+\int_0^x \frac{1}{v(u(y,t))}\partial_u f(p(y,t),u(y,t))\partial_y u(y,t)dy$$

$$+\frac{1}{v(u(0,t))}f(p(0,t),u(0,t)). \tag{16}$$

Using the change of variables

$$\mathscr{P}(x,t)=\partial_t p(x,t)-v(u(0,t))\partial_x p(x,t), \tag{17}$$

together with (2), (15) and (16) we derive the following equality

$$\mathscr{P}(x,t)=\int_0^x \frac{1}{v(u(y,t))}\partial_p f(p(y,t),u(y,t))\mathscr{P}(y,t)dy. \tag{18}$$

Computing the derivative of (18) with respect to x, one concludes that $\mathscr{P}(x,t)$ satisfies the following differential equation

$$\frac{\partial \mathscr{P}(x,t)}{\partial x}=\frac{1}{v(u(x,t))}\partial_p f(p(x,t),u(x,t))\mathscr{P}(x,t), \tag{19}$$

$$\mathscr{P}(0,t)=0, \tag{20}$$

for all $x\in[0,D]$, and thus, $\mathscr{P}(x,t)=0$, which is equivalent to

$$\partial_t p(x,t)-v(u(0,t))\partial_x p(x,t)=0. \tag{21}$$

Next, we differentiate the backstepping transformation (11) with respect to t and x in order to get that

$$\partial_t w(x,t)=\partial_t u(x,t)+\nabla\kappa\,(p(x,t))\,\partial_t p(x,t), \tag{22}$$

and

$$\partial_x w(x,t)=\partial_x u(x,t)+\nabla\kappa\,(p(x,t))\,\partial_x p(x,t). \tag{23}$$

From (2) and (21)–(23) the following holds

$$\partial_t w(x,t)-v(u(0,t))\partial_x w(x,t)=0. \tag{24}$$

Evaluating (11) at $x=0$ we get from (6) that

$$u(0,t)=w(0,t)+\kappa(X(t)), \tag{25}$$

which can be substituted into Eq. (24) to obtain (14). By direct verification from (11) for $x=0$ and (6) we derive the ODE dynamics (12). Setting $x=D$ into (11) we obtain the boundary condition (14) via (5).

Lemma 2. *The inverse of the infinite-dimensional backstepping transformation (11) is given by*

$$u(x,t)=w(x,t)+\kappa\,(\pi(x,t))\,, \tag{26}$$

where π is defined as

$$\pi(x,t)=X(t)+\int_0^x\Bigg[\frac{1}{v(w(y,t)+\kappa(\pi(y,t)))}$$

$$f\,(\pi(y,t),\kappa\,(\pi(y,t))+w(y,t))\Bigg]dy. \tag{27}$$

Proof. The inverse transformation (26) maps $w\mapsto u$ and is associated to the target system predictor, namely, (27) whereas the direct transformation (11), which maps $u\mapsto w$, is associated to the plant predictor, namely, (6). Even if the two predictor representations are not driven by the same input signals the following holds

$$p\equiv\pi. \tag{28}$$

Thus by direct verification, (26) is the inverse of (11). Clearly, taking the derivative of (27) with respect to x and t, it can be shown that relation (28) hold given that p and π satisfy the identical class of transport PDE (see (21)).

Lemma 3. *There exists a function $\mathscr{L}\in\mathcal{KL}$ such that*

$$|X(t)|+\sup_{x\in[0,D]}|w(x,t)|$$
$$\le\mathscr{L}\left(|X(0)|+\sup_{x\in[0,D]}|w_0(x)|,t\right), \tag{29}$$

for all $t\ge 0$.

Proof. Consider the following family of parameterized Lyapunov functions candidates for the target system's transport PDE (13)

$$L_{c,n}=\int_0^D e^{2ncx}w^{2n}(x,t)dx, \tag{30}$$

for any $c>0$ and positive integer n. The time derivative of $L_{c,n}(t)$ along (13) and (14) is written as

$$\begin{aligned}\dot L_{c,n}(t)&=\int_0^D e^{2ncx}2n\,(\partial_t w(x,t))\,w^{2n-1}(x,t)dx,\\&=2nv(w(0,t)+\kappa\,(X(t)))\\&\quad\times\int_0^D e^{2ncx}w(x,t)^{2n-1}\partial_x w(x,t)dx\\&=-v(w(0,t)+\kappa\,(X(t)))\\&\quad\times\left[w(0,t)^{2n}+2nc\int_0^D e^{2ncx}w(x,t)^{2n}dx\right].\end{aligned} \tag{31}$$

From Assumption 1 it holds that $v(\alpha)\ge\varepsilon$, for all $\alpha\in\mathbb{R}$, and thus, we get from (30), (31) that

$$\dot L_{c,n}(t)\le -2nc\varepsilon L_{c,n}(t). \tag{32}$$

Moreover, from (30) it follows that

$$\int_0^D|w(z,t)|^{2n}dz\le L_{c,n}(t)\le e^{2ncD}\int_0^D|w(z,t)|^{2n}dz, \tag{33}$$

for all $t\ge0$, $c>0$, and $n\in\mathbb{N}_+$. Integrating (32) and using (33) we get

$$\int_0^D|w(z,t)|^{2n}dz\le e^{-2nc\varepsilon(t-s)}e^{2ncD}\int_0^D|w(z,s)|^{2n}dz, \tag{34}$$

for all $t\ge s\ge 0$. From (34) we get

$$\left(\int_0^D|w(z,t)|^{2n}dz\right)^{\frac{1}{2n}}\le e^{-c\varepsilon(t-s)}e^{cD}\left(\int_0^D|w(z,s)|^{2n}dz\right)^{\frac{1}{2n}}. \tag{35}$$

Taking the limit as $n\to\infty$ and using the fact that

$$\lim_{n\to\infty}\left(\int_0^D|w(z,t)|^{2n}dz\right)^{\frac{1}{2n}}=\sup_{x\in[0,D]}|w(x,t)|\equiv\|w(t)\|_\infty \tag{36}$$

from (35) the following holds

$$\sup_{x\in[0,D]}|w(x,t)|\le e^{-c\varepsilon(t-s)}e^{cD}\left(\sup_{x\in[0,D]}|w(x,s)|\right), \tag{37}$$

for all $t\ge s\ge 0$. Based on Assumption 3, there exist some $\bar{\mathscr{L}}\in\mathcal{KL}$ and $\bar{\mathscr{K}}\in\mathcal{K}_\infty$ such that the solutions of (12) satisfy

$$|X(t)|\le\bar{\mathscr{L}}\big(|X(s)|,t-s\big)+\bar{\mathscr{K}}\left(\sup_{\tau\in[s,t]}|w(0,\tau)|\right), \tag{38}$$

for all $t\ge s\ge 0$. We perform the change of variables $s=\frac{t}{2}$ and rewrite (38) as

$$|X(t)|\le\bar{\mathscr{L}}\left(\left|X\left(\frac{t}{2}\right)\right|,\frac{t}{2}\right)+\bar{\mathscr{K}}\left(\sup_{\tau\in[\frac{t}{2},t]}|w(0,\tau)|\right). \tag{39}$$

The estimate of $\left|X\left(\frac{t}{2}\right)\right|$ follows by setting $s=0$ and substituting t by $\frac{t}{2}$ into (38). Hence, the following holds

$$\left|X\left(\frac{t}{2}\right)\right| \le \bar{\mathscr{L}}\left(|X(0)|, \frac{t}{2}\right) + \bar{\mathscr{K}}\left(\sup_{\tau\in[0,\frac{t}{2}]} |w(0,\tau)|\right). \tag{40}$$

From (37), we derive the estimates

$$\sup_{\tau\in[0,\frac{t}{2}]} \|w(\tau)\|_\infty \le e^{cD} \sup_{x\in[0,D]} |w_0(x)|, \tag{41}$$

$$\sup_{\tau\in[\frac{t}{2},t]} \|w(\tau)\|_\infty \le e^{-\frac{c\varepsilon}{2}t} e^{cD} \sup_{x\in[0,D]} |w_0(x)|. \tag{42}$$

Substituting (40) through (42) into (39) and using the fact that

$$|w(0,\tau)| \le \sup_{x\in[0,D]} |w(x,\tau)| \tag{43}$$

leads to (29) with

$$\begin{aligned}\mathscr{L}(r,s) = {}& \bar{\mathscr{L}}\left(\bar{\mathscr{L}}\left(r, \frac{s}{2}\right) + \bar{\mathscr{K}}\left(re^{cD}\right), \frac{s}{2}\right) \\ & + \bar{\mathscr{K}}\left(e^{-\frac{c\varepsilon}{2}s} re^{cD}\right) + e^{-c\varepsilon s} re^{cD}.\end{aligned} \tag{44}$$

Lemma 4. *There exists a class $\mathcal{K}_\infty$ function $\mathscr{K}_9$ such that*

$$\sup_{x\in[0,D]} |p(x,t)| \le \mathscr{K}_9\left(|X(t)| + \sup_{x\in[0,D]} |u(x,t)|\right), \tag{45}$$

for all $t \ge 0$.

Proof. Taking the derivative of (6) with respect to x, we get

$$\partial_x p(x,t) = \frac{1}{v(u(x,t))} f(p(x,t), u(x,t)), \tag{46}$$

with the initial condition

$$p(0,t) = X(t). \tag{47}$$

Knowing that $v(\alpha) > 0$, for all $\alpha \in \mathbb{R}$, and using (9) we derive the following inequality

$$\begin{aligned}&\frac{\partial\Theta(p(x,t))}{\partial p} \frac{1}{v(u(x,t))} f(p(x,t), u(x,t)) \\ &\le \frac{1}{v(u(x,t))}\Big(\Theta(p(x,t)) + \mathscr{K}_3(|u(x,t)|)\Big).\end{aligned} \tag{48}$$

Then, using (46) the following holds

$$\begin{aligned}\frac{\partial\Theta(p(x,t))}{\partial x} \le {}& \frac{1}{v(u(x,t))}\Theta(p(x,t)) \\ & + \frac{1}{v(u(x,t))}\mathscr{K}_3(|u(x,t)|),\end{aligned} \tag{49}$$

and with the help of (7), inequality (49) yields

$$\frac{\partial\Theta(p(x,t))}{\partial x} \le \frac{1}{\varepsilon}\Theta(p(x,t)) + \frac{1}{\varepsilon}\mathscr{K}_3(|u(x,t)|). \tag{50}$$

From (50) we obtain

$$\Theta(p(x,t)) \le e^{\frac{x}{\varepsilon}}\Theta(p(0,t)) + \frac{1}{\varepsilon}\int_0^x e^{\frac{x-y}{\varepsilon}}\mathscr{K}_3(|u(y,t)|)\,dy \tag{51}$$

by the comparison principle. Using (47) we derive

$$\begin{aligned}\Theta(p(x,t)) \le {}& e^{\frac{D}{\varepsilon}}\Theta(X(t)) + \left(e^{\frac{D}{\varepsilon}} - 1\right) \\ & \times \mathscr{K}_3\left(\sup_{x\in[0,D]} |u(x,t)|\right).\end{aligned} \tag{52}$$

Using (8) the following inequality holds

$$\begin{aligned}|p(x,t)| \le {}& \mathscr{K}_1^{-1}\Bigg(e^{\frac{D}{\varepsilon}}\mathscr{K}_2(|X(t)|) + \left(e^{\frac{D}{\varepsilon}} - 1\right) \\ & \times \mathscr{K}_3\left(\sup_{x\in[0,D]} |u(x,t)|\right)\Bigg), \quad \text{for all } x \in [0,D].\end{aligned} \tag{53}$$

Defining

$$\mathscr{K}_9(s) = \mathscr{K}_1^{-1}\left(e^{\frac{D}{\varepsilon}}\mathscr{K}_2(s) + \left(e^{\frac{D}{\varepsilon}} - 1\right)\mathscr{K}_3(s)\right), \tag{54}$$

completes the proof.

Lemma 5. *There exists a class $\mathcal{K}_\infty$ function $\mathscr{K}_{10}$ such that*

$$\begin{aligned}&\sup_{x\in[0,D]} |\pi(x,t)| \le \mathscr{K}_{10}\left(|X(t)| + \sup_{x\in[0,D]} |w(x,t)|\right), \\ &t \ge 0.\end{aligned} \tag{55}$$

Proof. Based on the input-to-state stability of $\dot{X} = f(X, \kappa(X) + \omega)$ with respect to ω, namely, Assumption 3, there exist a smooth function $\mathscr{F}(X) : \mathbb{R}^n \to \mathbb{R}_+$ and class $\mathcal{K}_\infty$ functions $\mathscr{K}_4$, $\mathscr{K}_5$, $\mathscr{K}_6$, $\mathscr{K}_7$ such that (see, e.g., (Sontag, 1995))

$$\mathscr{K}_4(|X(t)|) \le \mathscr{F}(X(t)) \le \mathscr{K}_5(|X(t)|), \tag{56}$$

$$\begin{aligned}&\frac{\partial\mathscr{F}(X(t))}{\partial X} f(X(t), \kappa(X(t)) + w(0,t)) \\ &\le -\mathscr{K}_6(|X(t)|) + \mathscr{K}_7(|w(0,t)|).\end{aligned} \tag{57}$$

Differentiating (27) with respect to x the following ODE for π is derived for all $x \in [0,D]$

$$\begin{aligned}\partial_x\pi(x,t) = {}& \frac{1}{v(w(x,t) + \kappa(\pi(x,t)))} \\ & \times f\big(\pi(x,t), \kappa(\pi(x,t)) + w(x,t)\big),\end{aligned} \tag{58}$$

$$\pi(0,t) = X(t). \tag{59}$$

From (57), the following holds

$$\begin{aligned}&\frac{\partial\mathscr{F}(\pi(x,t))}{\partial \pi} f(\pi(x,t), \kappa(\pi(x,t)) + w(0,t)) \\ &\le -\mathscr{K}_6(|\pi(x,t)|) + \mathscr{K}_7(|w(0,t)|).\end{aligned} \tag{60}$$

Multiplying both sides of (60) by the transport velocity we get

$$\begin{aligned}&\frac{\partial\mathscr{F}(\pi(x,t))}{\partial \pi} \frac{1}{v(w(x,t) + \kappa(\pi(x,t)))} \\ &\times f(\pi(x,t), \kappa(\pi(x,t)) + w(0,t)) \\ &\le \frac{1}{v(w(x,t) + \kappa(\pi(x,t)))}\left(-\mathscr{K}_6(|\pi(x,t)|) + \mathscr{K}_7(|w(0,t)|)\right).\end{aligned} \tag{61}$$

Using (58), the relation (61) leads to the following inequality

$$\frac{\partial\mathscr{F}(\pi(x,t))}{\partial x} \le \frac{1}{v(w(x,t) + \kappa(\pi(x,t)))}\mathscr{K}_7(|w(0,t)|). \tag{62}$$

From the boundness of the transport speed (7), it follows that

$$\frac{\partial\mathscr{F}(\pi(x,t))}{\partial x} \le \frac{1}{\varepsilon}\mathscr{K}_7(|w(0,t)|). \tag{63}$$

Hence, from (59) it follows that

$$\mathscr{F}(\pi(x,t)) \le \mathscr{F}(X(t)) + \frac{D}{\varepsilon}\mathscr{K}_7(|w(0,t)|). \tag{64}$$

From (64) and (56), we deduce the existence of a class $\mathcal{K}_\infty$ function $\mathscr{K}_{10}$ such that (55) holds.

Proof of Theorem 1. Assumption 3 implies the existence of a class $\mathcal{K}_\infty$ function $\mathscr{K}_{11}$ such that

$$|\kappa(\xi)| \leq \mathscr{K}_{11}(|\xi|). \tag{65}$$

From the backstepping transformation (11) defined in Lemma 1 we deduce the following inequality

$$\sup_{x\in[0,D]} |w(x,t)| \leq \sup_{x\in[0,D]} \left(|u(x,t)| + \mathscr{K}_{11}(|p(x,t)|)\right). \tag{66}$$

From the inverse backstepping transformation (26) given in Lemma 2 we get

$$\sup_{x\in[0,D]} |u(x,t)| \leq \sup_{x\in[0,D]} \left(|w(x,t)| + \mathscr{K}_{11}(|\pi(x,t)|)\right). \tag{67}$$

Then, from (45) and (55) defined in Lemmas 4 and 5, respectively, we obtain

$$\sup_{x\in[0,D]} |w(x,t)| \leq \sup_{x\in[0,D]} |u(x,t)| + \mathscr{K}_{11} \circ \mathscr{K}_9\Bigg(|X(t)| + \sup_{x\in[0,D]} |u(x,t)|\Bigg), \tag{68}$$

$$\sup_{x\in[0,D]} |u(x,t)| \leq \sup_{x\in[0,D]} |w(x,t)| + \mathscr{K}_{11} \circ \mathscr{K}_{10}\Bigg(|X(t)| + \sup_{x\in[0,D]} |w(x,t)|\Bigg). \tag{69}$$

From (68) and (69), there exist class $\mathcal{K}_\infty$ functions $\mathscr{K}_{12}$ and $\mathscr{K}_{13}$ such that

$$|X(t)| + \sup_{x\in[0,D]} |w(x,t)| \leq \mathscr{K}_{12}\left(|X(t)| + \sup_{x\in[0,D]} |u(x,t)|\right), \tag{70}$$

$$|X(t)| + \sup_{x\in[0,D]} |u(x,t)| \leq \mathscr{K}_{13}\left(|X(t)| + \sup_{x\in[0,D]} |w(x,t)|\right). \tag{71}$$

Combining (29) of Lemma 3 and (71) we conclude that

$$|X(t)| + \sup_{x\in[0,D]} |u(x,t)| \leq \mathscr{K}_{13}\left(\mathscr{L}\left(|X(0)| + \sup_{x\in[0,D]} |w_0(x)|, t\right)\right). \tag{72}$$

Using (70) we recover (10) with $\mathscr{L}_0(s) = \mathscr{K}_{13}\left(\mathscr{L}(\mathscr{K}_{12}(s))\right)$.

In order to prove the well-posedness of the closed-loop system consisting of (1)–(3) with the controller (5), (6), we first show the well-posedness of the target system (12)–(14). Consider first the following system

$$\dot{\bar{X}}(\tau) = \frac{1}{v\left(\bar{w}(0,\tau) + \kappa\left(\bar{X}(\tau)\right)\right)} \times f\left(\bar{X}(\tau), \kappa\left(\bar{X}(\tau) + \bar{w}(0,\tau)\right)\right) \tag{73}$$

$$\bar{w}_\tau(x,\tau) = \bar{w}_x(x,\tau) \tag{74}$$

$$\bar{w}(D,\tau) = 0. \tag{75}$$

for all $\tau \geq 0$ and $0 \leq x \leq D$, with an initial condition

$$\bar{X}(0) = X(0), \tag{76}$$

$$\bar{w}(x,0) = w_0(x). \tag{77}$$

The explicit solution that satisfies the PDE (74) and the boundary condition (75) (which can be viewed as a delay line with D time units delay and zero input) is given by

$$\bar{w}(x,\tau) = \begin{cases} w_0(x+\tau), & 0 \leq x+\tau \leq D \\ 0, & x+\tau > D. \end{cases} \tag{78}$$

The uniqueness of this solution follows from the well-known uniqueness of solutions to the transport PDE with constant transport speed (74) with boundary condition (75), see, e.g., (Coron, 2007).

Moreover, from the definition for p in (6) we get that

$$p_{0_x}(x) = \frac{1}{v\left(u_0(x)\right)} f\left(p_0(x), u_0(x)\right). \tag{79}$$

Thus, under Assumption 1 and using the fact that v and f are assumed to be continuously differentiable, one can conclude from the local Lipschitzness assumption on u_0 and the compatibility condition that $p_0(x)$ is continuously differentiable on $[0, D]$. Therefore, from the backstepping transformation (11), the regularity assumption on u_0, and the fact that κ is assumed to be continuously differentiable, one can conclude that $w_0(x)$ is locally Lipschitz on $[0, D]$. Hence, from (78) and the compatibility condition one can deduce that $\bar{w}(x,t)$ is locally Lipschitz on $[0,D] \times [0,\infty)$.

Moreover, for all $\tau > D$ it follows from (73), (78) that

$$\dot{\bar{X}}(\tau) = \frac{1}{v\left(\kappa\left(\bar{X}(\tau)\right)\right)} f\left(\bar{X}(\tau), \kappa\left(\bar{X}(\tau)\right)\right), \tag{80}$$

and thus, the regularity assumptions on f, v, and κ as well as (7), guarantee that there exists a unique solution $\bar{X}(\tau) \in C^1(D,\infty)$. Similarly, for all $\tau < D$, it follows from (73), (78) that

$$\dot{\bar{X}}(\tau) = \frac{1}{v\left(w_0(\tau) + \kappa\left(\bar{X}(\tau)\right)\right)} \times f\left(\bar{X}(\tau), \kappa\left(\bar{X}(\tau) + w_0(\tau)\right)\right), \tag{81}$$

and thus, using the fact that w_0 is locally Lipschitz we conclude that there exists a unique solution $\bar{X}(\tau) \in C^1[0, D)$. The compatibility condition guarantees that $\bar{X}$ is continuously differentiable also at $\tau = D$, and thus, $\bar{X}(\tau) \in C^1[0,\infty)$.

Define next the change of variables $\tau = \Phi(t)$, where

$$\Phi(t) = \int_0^t v\left(\bar{w}\left(0, \Phi(s)\right) + \kappa\left(\bar{X}\left(\Phi(s)\right)\right)\right) ds, \tag{82}$$

which implies that

$$\Phi'(t) = v\left(\bar{w}\left(0, \Phi(t)\right) + \kappa\left(\bar{X}\left(\Phi(t)\right)\right)\right), \tag{83}$$

with $\Phi(0) = 0$. From the regularity properties of the solutions to $\bar{w}$ and $\bar{X}$, it follows that the right-hand side of the latter ODE in Φ is locally Lipschitz with respect to Φ, and thus, there exists a unique $\Phi(t)$, which is now a continuously differentiable function for all $t \geq 0$ and, moreover, it satisfies $\Phi'(t) > 0$, for all $t \geq 0$. Thus, Φ is invertible with a continuously differentiable inverse function Φ^{-1}.

Then, setting

$$X(t) = \bar{X}\left(\Phi(t)\right), \tag{84}$$

$$w(x,t) = \bar{w}(x, \Phi(t)), \tag{85}$$

and using the facts that Φ is strictly increasing, that $\Phi(0) = 0$, and that

$$\lim_{t\to\infty} \Phi(t) = \infty, \tag{86}$$

one can conclude that there exists a unique solution $X(t) \in C^1[0,\infty)$ and that there exists a unique solution $w(x,t)$ such that $w(x,t)$ is locally Lipschitz on $[0,D]\times[0,\infty)$. The proof is completed noting that X satisfies (12) and that w satisfies (13), (14), facts that follow from (73)–(75), the definition of $\Phi(t)$, and definitions $X(t) = \bar{X}\left(\Phi(t)\right)$, $w(x,t) = \bar{w}(x, \Phi(t))$.

Using the equivalence between the signals $p(x,t)$ and $\pi(x,t)$ stated in (28), it can be deduced from (18) that the π-system satisfies the following PDE

$$\partial_t \pi(x,t) = v(w(0,t) + \kappa(X(t)))\partial_x \pi(x,t), \tag{87}$$

$$\pi(0,t) = X(t). \tag{88}$$

Defining the characteristic curves parameterized by some variable τ and expressing the total derivative of $\pi(x(\tau), t(\tau))$, the solution of the transport PDE (87) compatible with the boundary condition (88) is written as

$$\pi(x,t) = X\left(\Phi^{-1}(x+\Phi(t))\right). \tag{89}$$

The existence and uniqueness of $(X(t), \Phi(t)) \in C^1[0,\infty)$ ensures that $\pi(x,t)$ is continuously differentiable on $[0,D]\times[0,\infty)$, and thus, from the inverse backstepping transformation (26) and the local Lipschitzness of $w(x,t)$ we get the local Lipschitzness of $u(x,t)$ on $[0,D]\times[0,\infty)$.

4. Equivalent representation using a delay system

4.1. Relation to a nonlinear system with delayed-input-dependent input delay

An alternative representation of the cascade system (1)–(3), which incorporates a boundary-value-dependent propagation speed is offered in this section. We recast the original problem into a delay system framework by solving the transport PDE (2), (3) with the method of characteristics. The resulting system is a delayed-input-dependent input delay system, which can be explained by the fact that the propagation speed of the PDE, namely, $v(u(0,t))$ is itself dependent on the delayed boundary input $U(t) = u(D,t)$. The method of characteristics is used first in order to solve the transport PDE (2). Defining the characteristic curves parameterized by some variable τ, the distributed state of the PDE (2), namely, $u(x,t)$, can be described by $u(x(\tau), t(\tau))$ whose total derivative is written as

$$\frac{du(x(\tau),t(\tau))}{d\tau} = \frac{\partial u}{\partial t}\frac{dt}{d\tau} + \frac{\partial u}{\partial x}\frac{dx}{d\tau}. \tag{90}$$

Matching (90) to the original PDE (2) the following set of ODEs are derived

$$\frac{dt(\tau)}{d\tau} = 1, \tag{91}$$

$$\frac{dx(\tau)}{d\tau} = -v\left(u(0,t(\tau))\right), \tag{92}$$

$$\frac{du(x(\tau),t(\tau))}{d\tau} = 0. \tag{93}$$

The solution to (91)–(93) corresponds to the constant solution along the characteristics lines $(x(\tau), t(\tau))$. Integration of the ODEs (91) and (92) yields the characteristic curves of the PDE (2) given as

$$t(\tau) = t_0 + \tau, \quad t(0) = t_0 \tag{94}$$

$$x(\tau) = \int_0^\tau \frac{dx(\lambda)}{d\lambda}\,d\lambda + x(0) \tag{95}$$

$$= -\int_0^\tau v\left(u(0,t_0+\lambda)\right)d\lambda + D, \quad x(0) = D. \tag{96}$$

Now, we define the primitive function of the variable transport velocity as

$$\Phi_u(t) = \int_0^t v\left(u(0,\lambda)\right)d\lambda. \tag{97}$$

Since the transport velocity v is assumed to be strictly positive, the function $\Phi_u(t)$ is a monotonically increasing function and defines a bijective mapping between time and space. The subscript $_u$ denotes the delayed-input dependence of the function $\Phi_u(t)$. By combining (96) and (97) we derive the following relation

$$x(\tau) = \Phi_u(t_0) - \Phi_u(t_0+\tau) + D. \tag{98}$$

By considering the characteristic curves with $x(\tau) = 0$ at time $t = t_0 + \tau$ and defining the time delay $R_u(t) = t - t_0$ we get

$$D = \Phi_u(t) - \Phi_u(t - R_u(t)). \tag{99}$$

It is clear that such t always exists (and is unique for a given t_0) as $\Phi_u(t)$ is strictly increasing and from (7) (see Assumption 1)

$$\lim_{t\to\infty} \Phi_u(t) = \infty. \tag{100}$$

From (93) we know that the solution of the transport PDE (2) is constant along the characteristic curves. Thus,

$$u(0,t) = u(D, t - R_u(t)) = U(t - R_u(t)). \tag{101}$$

Consequently, using (99) and (101), the original cascade system (1)–(3) is reduced to a nonlinear system with an implicitly defined delayed-input-dependent input delay, which is written as

$$\dot X(t) = f\left(X(t), U(\phi(t))\right) \tag{102}$$

$$\phi(t) = t - R_u(t) \tag{103}$$

$$D = \int_{\phi(t)}^t v(U(\phi(\lambda)))d\lambda. \tag{104}$$

In summary, the delay function is computed considering the time that the input signal, $U(t) = u(D,t)$, needs to travel from the boundary $x = D$ to the boundary $x = 0$. Since the transport velocity depends on the uncontrolled boundary value dynamics, namely, $u(0,t)$ (see (2) and (3)), the delay function itself is dependent on the boundary value of $u(0,t)$. Such value is computed in Eq. (101) by using the well-known method of characteristics as an implicitly given function $R_u(t) = t - t_0$. The control signal $U(t) = u(D,t)$ can be computed from the signal $u(0,t)$ considering that the signal that enters the boundary $x = D$ needs $R_u(t)$ units of time to reach the boundary $x = 0$. Thus, for any given time t the equality (101) holds.

4.2. Predictor-feedback for the equivalent delay system

The predictor feedback control law for system (102) is

$$U(t) = \kappa(P(t)), \tag{105}$$

where $\kappa(X)$ is the nominal feedback control law for the delay free plant $\dot X(t) = f(X(t), U(t))$ *and*

$$P(t) = X(t) + \int_{\phi(t)}^t \frac{v(U(\phi(\theta)))}{v(U(\theta))} f\left(P(\theta), U(\theta)\right)d\theta, \tag{106}$$

with the initial condition

$$P(\theta) = X(0) + \int_{\phi(0)}^\theta \frac{v(U(\phi(s)))}{v(U(s))} f\left(P(s), U(s)\right)ds, \tag{107}$$

for all $\phi(0) \le \theta \le 0$. The predictor signal (106) is derived by defining the prediction time from the implicitly given delay function (104) as

$$\Sigma(t) = \phi^{-1}(t). \tag{108}$$

Thus, the following implicit relation for Σ holds

$$D = \int_t^{\Sigma(t)} v(U(\phi(\lambda)))d\lambda. \tag{109}$$

The time derivative of (109) is expressed as

$$\dot\Sigma(t) v\left(U(t)\right) - v\left(U(\phi(t))\right) = 0, \tag{110}$$

which leads to

$$\dot{\Sigma}(t) = \frac{v(U(\phi(t)))}{v\,(U\,(t))}. \tag{111}$$

Substitution of $t = \Sigma(\theta)$, $\theta \in [\phi(t), t]$, in (102) enables one to derive the following differential equation

$$\frac{d}{d\theta}(X(\Sigma(\theta))) = \dot{\Sigma}(\theta)f(X\,(\Sigma(\theta))\,,U(\theta)). \tag{112}$$

Using (111) one arrives at

$$\frac{d}{d\theta}(X(\Sigma(\theta))) = \frac{v(U(\phi(\theta)))}{v\,(U\,(\theta))}f\,(X\,(\Sigma(\theta))\,,U(\theta))\,. \tag{113}$$

Defining the predictor signal as

$$P(\theta) = X\,(\Sigma(\theta)) \tag{114}$$

and integrating (113) over $[\phi(t), t]$ the predictor (106) with initial condition (107) is derived. The relation between the states $(P(\theta), U(\theta))$, $\theta \in [\phi(t), t]$ and $(p(x, t), u(x, t))$, $x \in [0, D]$ is provided in Appendix.

5. Application to production systems

5.1. Motivation: Control of a PDE model of a network of suppliers

Production and distribution systems, which convey a large number of parts, are often used in various manufacturing and logistics processes such as electronic and automobile industries. A crucial factor in supply chain networks is the ability to produce uniformly, high-quality final parts at a high rate while avoiding machine break-down and operation variations that may impose a reconfiguration of the entire process. The dynamic nature of such systems depends strongly on the work in progress (WIP) and the frequency of new product entering the system.

Several modeling approaches are utilized to capture the evolution in time of supply chains' networks. These dynamical representations are often exploited for parts flow control, fault detection, system's diagnostics, and tasks planning and management purposes. The concept of agent-based models (Gjerdrum, Shah, & Papageorgiou, 2001; Swaminathan, Smith, & Sadeh, 1998), which was initially developed for the control of distributed systems, is suitable to describe the time evolution of production chains that involve the transformation of raw materials into intermediate and finished products with a distribution system of the final products. For large scale systems, discrete event representation (Kleijnen, 2005; Tako & Robinson, 2012; Terzi & Cavalieri, 2004) is well-known to be exceedingly expensive to maintain and computationally difficult to handle. These approaches fail in reflecting the production system dynamics as a whole and lead to machine breakdown and delay of parts arrival. An exhaustive discussion on such problematics can be found in Sun and Dong (2008) and the references therein.

Therefore, PDE representations (Armbruster, Degond, & Ringhofer, 2006; Borsche et al., 2010; Coron & Wang, 2013; Herty et al., 2007; Marca, Armbruster, Herty, & Ringhofer, 2010; Shang & Wang, 2011; Sun & Dong, 2008) are more efficient computationally, more robust and more accurate for predicting the WIP as well as the outflux behaviors in production systems. In this section, we present a PDE model of the dynamics of a production system consisting of a chain of outgoing suppliers connected to a queue in front of it. The supply chain is described by a PDE with boundary-value-dependent propagation speed (Sun & Dong, 2008) and the load of goods stored in the queue is modeled by a nonlinear ODE whose input is given by the value of the PDE at the exit of the production line (Borsche et al., 2010; Papadopoulos & Heavey, 1996).

5.2. PDE model of a supply chain system with a finite buffer size

The conservation law within a single node and the constitutive relations between velocity and parts density for production systems depend on the flow of parts entering the node at $x = D$ (raw materials) and exiting at $x = 0$ as finished products and losses. Here the spatial variable x is an artificial continuous index of the suppliers. Denoting $\rho(x, t)$ the density of parts at stage x and time t and $\omega(x, t)$ the velocity of the product movement through the node, the following PDE model which expresses the conservation of parts in the node is defined in Armbruster et al. (2006) and Sun and Dong (2008)

$$\partial_t \rho(x, t) - \partial_x\,(\omega(x, t)\rho(x, t)) = 0. \tag{115}$$

The transport equation (115) is obtained expressing the mass conservation laws into the system if the yield loss on stages is neglected (no production or loss of material during the production process). As in Sun and Dong (2008), we assume that the speed ω is a function of parts density ρ and define the following relation

$$\omega(x, t) = v(\rho(x, t))\Big|_{x=0} = \frac{1}{P(1 + \rho(x, t))}\Big|_{x=0}, \tag{116}$$

where P is the processing time. Assuming that the exit density obeys a uniform law for each stage $x \in (0, D)$, (115) is reduced to

$$\partial_t \rho(x, t) - \left(\frac{1}{P(1 + \rho(0, t))}\right)\partial_x \rho(x, t) = 0. \tag{117}$$

The nonlocal term in (117) suggests that the WIP depends nonlinearly upon only the output density at the current time and indicates the full impact of the overall loading of the system on the motion at the beginning of the production line. The density $\rho(x, t)$ is a physical quantity which remains non-negative and bounded. One should note that the velocity $v(\rho(0, t))$ is positive and becomes zero only if the factory breaks down completely.

The initial condition is defined as $\rho(x, 0) = \rho_0(x)$, The conservation law imposes the following boundary condition at the entrance of the production line

$$\omega(D, t)\rho(D, t) = F(t). \tag{118}$$

Knowing that the propagation velocity, which is defined in (116), remains spatially independent enables one to write the boundary condition (118) as follows

$$\frac{1}{P(1 + \rho(0, t))}U(t) = F(t), \tag{119}$$

where F is the control input and U is the "virtual" input to be designed based on the infinite-dimensional predictor feedback (5), (6).

5.3. Queue modeling of the finite buffer

In this section, we present the dynamical model of the queue in front of the production line. We represent the queue buffer occupancy by defining the load of goods, Q, stored in the queue and imposing the conservation of mass (Borsche et al., 2010; Herty et al., 2007). Hence,

$$\dot{Q}(t) = v^{\text{in}}(\rho(0, t)) - v^{\text{out}}(Q(t)), \tag{120}$$

where the influx in the buffer is defined as

$$v^{\text{in}}(\rho(0, t)) = \alpha v(\rho(0, t))\rho(0, t), \tag{121}$$

and α is the connectivity coefficient, which denotes the fraction of materials flux flowing from the final stage to the buffer when the

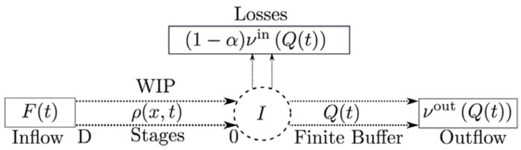


Fig. 1. Non-re-entrant manufacturing chain with uniform propagation velocity. The symbol I denotes the distribution structure.

rate of losses is assumed to be known and equal to $1-\alpha$ (see Fig. 1). The queue is characterized by a maximum processing capacity $Q_{\max}$ and a maximum service rate μ. Therefore, we consider that the flux at the output is given as[1]

$$\nu^{\text{out}}(Q(t)) = \min\left(Q(t), \mu\right). \tag{122}$$

Finally, substituting (116), (121) and (122) into (120), we obtain the following ODE

$$\dot{Q}(t) = \frac{\alpha\rho(0,t)}{P(1+\rho(0,t))} - \min\left(Q(t), \mu\right). \tag{123}$$

For the model described in Eqs. (117), (123) with the boundary condition (118), the control objective is to stabilize the queue Q at the desired equilibrium value $Q^\star$.

5.4. "Bang–Bang" control of the delay-free plant

In order to ensure a fast stabilization of the production system to the desired set point $Q^\star$ satisfying $0 \le Q^\star < \mu$, we employ a piecewise exponential "bang–bang" control law (Diagne & Krstic, 2015). The delay-free plant is defined as

$$\dot{Q}(t) = \frac{\alpha U(t)}{P(1+U(t))} - \min\left(Q(t), \mu\right), \tag{124}$$

where $U(t)$ is the virtual nominal input. System (124) admits an open-loop control law defined as

$$b(Q^\star) = \frac{P\min\left(Q^\star, \mu\right)}{\alpha - P\min\left(Q^\star, \mu\right)}. \tag{125}$$

Since $Q^\star < \mu$, we have that

$$b(Q^\star) = \frac{PQ^\star}{\alpha - PQ^\star}. \tag{126}$$

From the non-negativeness of the density function $\rho(x,t)$ and the propagation speed $v(\rho(0,t))$, we deduce that the setpoint control of the queuing model is a non-negative quantity imposing the following restriction

$$0 \le Q^\star < \min\left\{\frac{\alpha}{P}, \mu\right\}. \tag{127}$$

The stability of the system (124) under the setpoint control law (126) is proved below using the classical quadratic Lyapunov function $V = \frac{1}{2}\tilde{Q}^2$, where $\tilde{Q} = Q - Q^\star$. The time derivative of this Lyapunov function along the solution to (124) subject to the control law (126) is written as follows:

$$\dot{V} = \tilde{Q}\left[\frac{\alpha b(Q^\star)}{P(1+b(Q^\star))} - \min\left(\tilde{Q}+Q^\star, \mu\right)\right], \tag{128}$$

and equivalently

$$\dot{V} = \tilde{Q}M(\tilde{Q}, Q^\star), \tag{129}$$

where

$$M(\tilde{Q}, Q^\star) = Q^\star - \min\left(\tilde{Q}+Q^\star, \mu\right). \tag{130}$$

From (130) one can deduce that

$$\text{sign}\{M(\tilde{Q}, Q^\star)\} = -\text{sign}\{\tilde{Q}\}, \tag{131}$$

which immediately implies that $\dot{V} < 0$, for all $\tilde{Q} \ne 0$, and thereby, induces the asymptotic stability of (124).

5.5. Piecewise exponential "bang–bang" control law

The piecewise exponential "bang–bang" control law which was introduced in Diagne and Krstic (2015) enables one to stabilize the closed-loop delay-free plant to the desired setpoint. We refer the reader to Diagne and Krstic (2015) in which such a control approach is widely discussed. Consider the feedback law

$$\begin{aligned} B(Q(t), Q^\star) = {} & B_{\text{l}}\big(Q(t), Q^\star\big)\mathcal{H}\big(Q^\star - Q(t)\big) \\ & + B_{\text{r}}\big(Q(t), Q^\star\big)\mathcal{H}\big(Q(t) - Q^\star\big), \end{aligned} \tag{132}$$

where $\mathcal{H}$ is the Heaviside function and the functions B_{l} and B_{r} are given by

$$\begin{aligned} B_{\text{l}}\big(Q(t), Q^\star\big) = {} & b(Q^\star) \\ & + \big(B_{\max} - b(Q^\star)\big)\frac{1 - e^{\Lambda_{\text{l}}(Q^\star)(Q(t)-Q^\star)}}{1 - e^{-\Lambda_{\text{l}}(Q^\star)(Q^\star)}}, \end{aligned} \tag{133}$$

$$B_{\text{r}}(Q(t), Q^\star) = b(Q^\star) - b(Q^\star)\frac{1 - e^{-\Lambda_{\text{r}}(Q^\star)(Q-Q^\star)}}{1 - e^{-\Lambda_{\text{r}}(Q^\star)(Q_{\max}-Q^\star)}}, \tag{134}$$

for $Q \ge Q^\star$. Here, B is the input density and $B_{\max}$ and $Q_{\max}$ are the maximum value of the input density and capacity of the queue, respectively. In order to guarantee a well-running process, these maximum values are chosen to satisfy

$$B_{\max} \le \frac{\min\{Q_{\max}, \mu\}}{\alpha - P\min\{Q_{\max}, \mu\}}, \tag{135}$$

$$\alpha - P\min\{Q_{\max}, \mu\} > 0. \tag{136}$$

The functions $\Lambda_{\text{l}}(Q^\star) > 0$ and $\Lambda_{\text{r}}(Q^\star) > 0$ are the gains of the left and right exponential control law defined as (133) and (134), respectively. These gains are uniquely determined to ensure the continuous differentiability of the extended control law (132). We define the setpoint slope functions of (133) and (134), namely, $S(Q^\star)$ as the sole design parameter of the controller, which satisfies the following relations

$$S(Q^\star) = \frac{\Lambda_{\text{l}}(Q^\star)\big(B_{\max} - b(Q^\star)\big)}{1 - e^{-\Lambda_{\text{l}}(Q^\star)Q^\star}}, \tag{137}$$

$$S(Q^\star) = \frac{\Lambda_{\text{r}}(Q^\star)b\big(Q^\star\big)}{1 - e^{-\Lambda_{\text{r}}(Q^\star)(Q_{\max}-Q^\star)}}. \tag{138}$$

The right hand sides of (137) and (138) are deduced differentiating (133) and (134) with respect to the state Q for $Q = Q^\star$. The gains $\Lambda_{\text{l}}(Q^\star)$ and $\Lambda_{\text{r}}(Q^\star)$ are computed as the unique strictly positive solutions of the transcendental equations (137) and (138), selecting $S(Q^\star)$ above the following minimal set (see Diagne and Krstic, 2015 for details)

$$S_{\min}(Q^\star) = \max\left\{\frac{(B_{\max} - b(Q^\star))}{Q^\star}; \frac{b(Q^\star)}{Q_{\max} - Q^\star}\right\}. \tag{139}$$

Proposition 1. *For any setpoint $Q^\star \in \left[0, \min\left\{Q_{\max}, \mu, \frac{\alpha}{P}\right\}\right)$ and for any chosen setpoint slope $S(Q^\star) \in \mathbb{R}$ satisfying $S(Q^\star) \ge S_{\min}(Q^\star)$,*

[1] The outflow is expressed using the relaxed formulation $\nu^{\text{out}}(Q(t)) = \min\left(\frac{Q(t)}{\varepsilon}, \mu\right)$ as presented in Armbruster, de Beer, Freitag, Jagalski, and Ringhofer (2006) (see also (Borsche et al., 2010)). For sake of simplicity, we consider the case for which the small smoothing parameter ε is set to $\varepsilon = 1$ but the control approach is easily applicable to the case of non-unity relaxation coefficient, namely, when $\varepsilon \ne 1$.

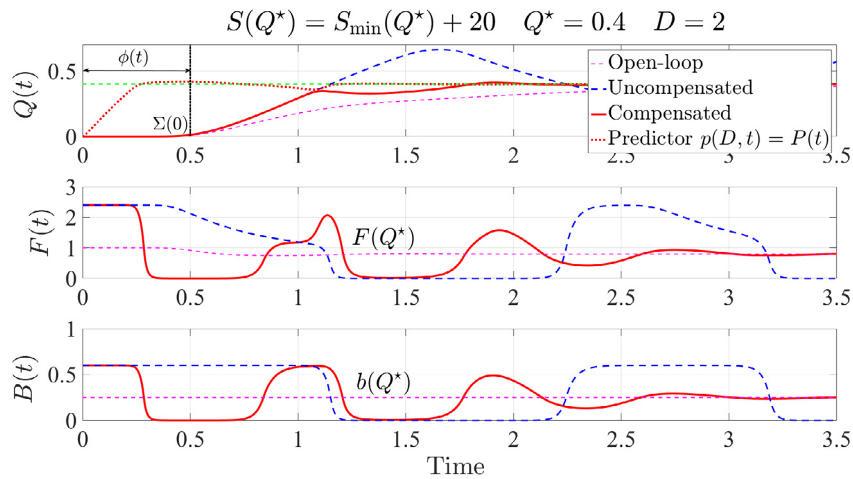


Fig. 2. Delay compensated "bang–bang" control of the queue capacity.

where $S_{\min}(Q^\star)$ is given by (139), taking the control gains (Λ_l, Λ_r) as solutions of [2]

$$\Lambda_l(B_{\max} - b(Q^\star)) - S(Q^\star)(1 - e^{-\Lambda_l Q^\star}) = 0, \quad (140)$$

$$\Lambda_r b(Q^\star) - S(Q^\star)(1 - e^{-\Lambda_r(Q_{\max} - Q^\star)}) = 0, \quad (141)$$

the closed-loop system consisting of (124) with an initial condition $Q_0 \in \left[0, \min\left\{Q_{\max}, \mu, \frac{\alpha}{P}\right\}\right)$ and control law (132) is asymptotically stable at $Q = Q^\star$.

Proof of Proposition 1. Consider the error system:

$$\dot{\tilde{Q}}(t) = \frac{\alpha U(t)}{P(1 + U(t))} - \min\left(\tilde{Q}(t) + Q^\star, \mu\right), \quad (142)$$

where $\tilde{Q} = Q(t) - Q^\star$, and the Lyapunov function

$$V(\tilde{Q}) = |\tilde{Q}|. \quad (143)$$

It follows that

$$\dot{V} = \dot{\tilde{Q}}\operatorname{sign}\{\tilde{Q}\}. \quad (144)$$

Hence,

$$\nabla_U \dot{V} = \frac{\alpha}{P(1 + U)^2}\operatorname{sign}\{\tilde{Q}\}, \quad (145)$$

Thus, $\dot{V}$ is a monotonically increasing or decreasing function of the input U. The asymptotic stability of system (142) can be deduced using the properties of the "bang–bang" controller. Setting $U = B(\tilde{Q}, Q^\star)$ in (145) the following hold.

- For all $\tilde{Q} < 0$, it holds that $B(\tilde{Q}, Q^\star) \in (b(Q^\star), B_{\max}]$ and that $\dot{V}$ is a decreasing function of U. When $B(\tilde{Q}, Q^\star) = b(Q^\star)$

$$\dot{V} = -\left[\frac{\alpha b(Q^\star)}{P(1 + b(Q^\star))} - \min(Q^\star + \tilde{Q}, \mu)\right]. \quad (146)$$

  Knowing $\tilde{Q} < 0$ and $Q^\star < \mu$, the following holds

$$\min(Q^\star + \tilde{Q}, \mu) < Q^\star. \quad (147)$$

  Thus, from (146), we deduce

$$\dot{V} < -\left[\frac{\alpha b(Q^\star)}{P(1 + b(Q^\star))} - Q^\star\right]. \quad (148)$$

  Hence, $\dot{V} < 0$, for all $\tilde{Q} < 0$ follows from the fact that $\dot{V}$ is a decreasing function of U.
- Similarly, for all $\tilde{Q} > 0$, $B(\tilde{Q}, Q^\star) \in [0, b(Q^\star))$. Knowing that $\dot{V}$ is an increasing function of U with $\dot{V} < 0$ when $B(\tilde{Q}, Q^\star) = b(Q^\star)$, we conclude that $\dot{V} < 0$, for all $\tilde{Q} > 0$.

Finally, the piecewise exponential feedback control law (132) imposes $\dot{V} < 0$, for all $\tilde{Q} \neq Q^\star$, which ensures the asymptotic stability of the error system (142).

[2] Analogously to Diagne and Krstic (2015), relations (140) and (141) are derived stating that $S(Q^\star) = -\frac{d}{dQ}B_l(Q(t), Q^\star)|_{Q=Q^\star} = -\frac{d}{dQ}B_r(Q(t), Q^\star)|_{Q=Q^\star}$, where $B_l(Q(t), Q^\star)$, and $B_r(Q(t), Q^\star)$ are defined in (133) and (134), respectively.

5.6. Predictor-feedback control design

From (5) and (6), the predictor-feedback control law of the production line is written as

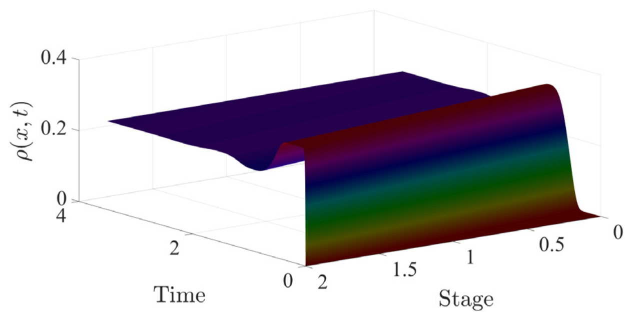
$$F(t) = \frac{1}{P(1 + \rho(0, t))}B(p(D, t), Q^\star), \quad (149)$$

$$p(x, t) = X(t) + \int_0^x P(1 + \rho(y, t)) \times \left(\frac{\alpha\rho(y, t)}{P(1 + \rho(y, t))} - \min(p(y, t), \mu)\right)dy. \quad (150)$$

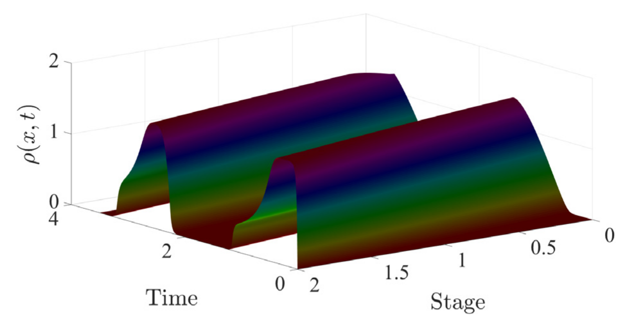
6. Simulation results

Simulations are performed in order to stabilize the queue buffer occupancy to a setpoint value $Q^\star = 0.4$ whereas the maximum capacity is set to $\mu = 0.8$ and the process time to $P = 0.25$. The maximum capacity of the queue is set to $Q_{\max} = 1$ and the maximum value of the input to $B_{\max} = 0.6$ with a distribution coefficient $\alpha = 0.5$. Physically, the queue is located at the stage $x = 0$ and the supplier device at the stage $x = 2$. The factory starts with zero parts density, $\rho_0(x) = 0$, at the initial time. In order to implement the delayed-input-dependent input delay "bang–bang" compensator, the value of the slope function is set to $S(Q^\star) = S_{\min}(Q^\star) + 20$.

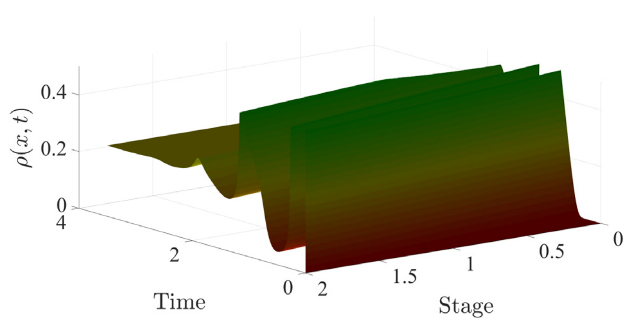
The dynamics of the queue buffer occupancy $Q(t)$, the boundary control law $F(t)$ and the nominal compensated "bang–bang" control law $B(p(D, t))$ are presented. Moreover, the evolution in time and space of the part density $\rho(x, t)$ and the predictor state $p(x, t)$ are simulated using a finite volume setup. As shown in Figs. 2 and 3, the uncompensated input leads to oscillatory response and the compensated controller allows faster convergence than the open loop control. One should point out that Karafyllis (2011) and Karafyllis and Krstic (2014) extensively discuss the



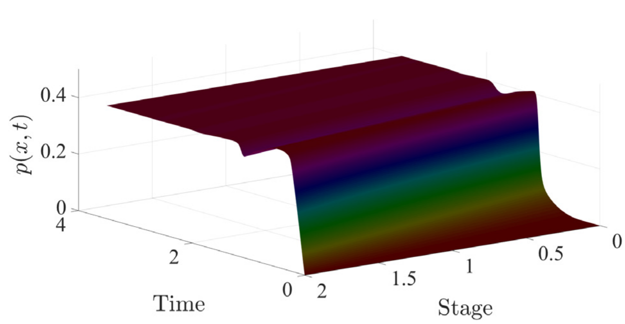
(a) Open-loop $\rho(x, t)$.



(b) Uncompensated $\rho(x, t)$.



(c) Compensated $\rho(x, t)$.



(d) Predictor $p(x, t)$.

Fig. 3. Evolution of the distributed variables.

implementation issue of predictor-feedback and offer various numerical schemes for computation of predictor-feedback laws.

7. Concluding remarks

In this paper, we develop an infinite-dimensional predictor-feedback control law which enables one to compensate, for nonlinear systems, actuator dynamics governed by a transport PDE with boundary-value-dependent propagation velocity. In particular, the actuator dynamics depend on the boundary value of the actuator state anti-collocated to the actuation or sensing mechanism introducing a delay that depends on the delayed input signal. Our predictor-feedback controller guarantees a global asymptotic stability of the coupled system. The feasibility of the proposed controller is demonstrated on a model of a production system with finite buffer, using the recently developed "bang–bang" nominal controller.

Appendix. Relation between (P, U) and (p, u)

Define the infinite-dimensional representation of the actuator state as

$$u(x,t) = U\left(\phi\left(\Phi_u^{-1}\left(x + \Phi_u(t)\right)\right)\right). \tag{A.1}$$

Using (99) we get

$$D = \Phi_u(\Sigma(t)) - \Phi_u(t). \tag{A.2}$$

From (A.1) we deduce

$$u(0,t) = U\left(\phi\left(\Phi_u^{-1}\left(\Phi_u(t)\right)\right)\right) = U(\phi(t)). \tag{A.3}$$

Using (A.2) and (A.1) the following holds

$$u(D,t) = U\left(\phi\left(\Sigma(t)\right)\right) = U(t). \tag{A.4}$$

Taking the derivative of (A.1) with respect to x and t, and knowing that

$$\frac{d\Phi_u(t)}{dt} = v\left(u(0,t)\right), \tag{A.5}$$

from (97), the actuator dynamics (2) are obtained.

Define next the infinite-dimensional representation of the predictor state as

$$p(x,t) = X\left(\Phi_u^{-1}\left(x + \Phi_u(t)\right)\right). \tag{A.6}$$

It follows that

$$p(0,t) = X\left(t\right). \tag{A.7}$$

Moreover, using (A.2) we get

$$p(D,t) = X\left(\Sigma(t)\right). \tag{A.8}$$

Taking the derivative of (A.6) with respect to x and t we get

$$\partial_x p(x,t) = \frac{1}{v(u(0,t))}\partial_t p(x,t). \tag{A.9}$$

Integrating (A.9) over $[0, x]$, we arrive at

$$p(x,t) = X(t) + \int_0^x \frac{1}{v(u(0,t))}\partial_t p(y,t)dy, \tag{A.10}$$

where the partial derivative of $p(x, t)$ with respect to time is written as

$$\begin{aligned}\partial_t p(x,t) = {} & v(u(0,t))\left(\Phi_u^{-1}\right)'\left(x + \Phi_u(t)\right)\\ & \times f\Bigg(X\left(\Phi_u^{-1}\left(x + \Phi_u(t)\right)\right),\\ & U\left(\phi\left(\Phi_u^{-1}\left(x + \Phi_u(t)\right)\right)\right)\Bigg),\end{aligned} \tag{A.11}$$

with the help of (102) and (97). Substituting (A.11) into (A.10) and employing relations (A.1) and (A.6) we get the following equality

$$p(x,t) = X(t) + \int_0^x \left(\Phi_u^{-1}\right)' (y + \Phi_u(t)) f\left(p(y,t), u(y,t)\right) dy. \tag{A.12}$$

Next, using (97) and (A.3), the following holds

$$x + \Phi_u(t) = \int_0^{\Phi_u^{-1}(x+\Phi_u(t))} v\left(U\phi(\lambda)\right) d\lambda. \tag{A.13}$$

Taking the derivative of both sides of (A.13) with respect to x and using (A.1) we deduce the following relation

$$\left(\Phi_u^{-1}\right)' (x + \Phi_u(t)) = \frac{1}{v\left(u(x,t)\right)}. \tag{A.14}$$

Finally, substituting (A.14) into (A.12), the predictor (6) is obtained.

Integrating (A.14) over the spatial domain $[0, x]$ we arrive at

$$\Phi_u^{-1}\left(x + \Phi_u(t)\right) = t + \int_0^x \frac{1}{v\left(u(y,t)\right)} dy. \tag{A.15}$$

Hence, $\sigma(x,t) = \Phi_u^{-1}\left(x + \Phi_u(t)\right)$.

Moreover, using (A.8) and (114) the equivalence between $p(D,t)$ and $P(t)$ could be derived. Analogously, from (A.2) we get $\Sigma(t) = \Phi_u^{-1}\left(D + \Phi_u(t)\right)$, which implies the equivalence between $\sigma(D,t)$ and $\Sigma(t)$.

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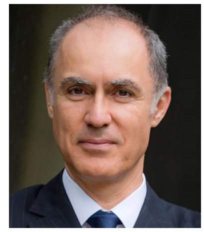

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