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Delay-adaptive feedback for linear feedforward systems

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ABSTRACT

Predictor techniques are an indispensable part of the control design toolbox for plants with input and state delays of significant size. Yet, they suffer from sensitivity to the design values. Explicit feedback laws were recently introduced by Jankovic for a class of feedforward linear systems with simultaneous state and input delays. For the case where the delays are of unknown length, using the certainty equivalence principle, we design a Lyapunov-based adaptive controller, which achieves global stability and regulation, for arbitrary initial estimates for the delays. We consider a two-block sub-class of linear feedforward systems. A generalization to the n-block case involves a recursive application of the same techniques.

1. Introduction

The design of stabilizing controllers for systems with delays continues to be an active area of research. Controllers for both linear and nonlinear systems exist in the literature, many of which are based on predictor-like techniques [1–9,24–38]. However, systems with simultaneous input and state delay remain a challenge, even for linear systems [10,2–4,11–13].

An even more challenging problem is the adaptive control of systems with simultaneous input and state delays. From the practical point of view, controllers for delay systems should be robust to parametric uncertainties, including plant parameters and delays. The importance of designing robust controllers when the delays are the unknown parameters was highlighted in the control problems considered in [14] and [15]. On the other hand, since there already exists a rich literature for the control of time delay systems, adaptive control schemes that are based on existing control techniques are of interest. Since many control schemes are based on predictor-like techniques, which are known to be very sensitive to delay uncertainties [16], designing adaptive versions of these control schemes is crucial for making them usable in scenarios with uncertain delays.

Adaptive control schemes can be found in [17,18]. Yet, the adaptive control problem when the delays are the unknown parameters had not been solved until recently with the works in [19,20]. In [19] the problem of designing an adaptive control scheme for a linear system with unknown input delay is solved, and in [20] the result is extended to also incorporate unknown plant parameters. The aforementioned designs are based on predictor feedback together with tools that come from the adaptive control of parabolic PDEs [21].

In this paper we develop a delay-adaptive version of the design introduced by Jankovic in [4] for linear feedforward systems with simultaneous state and input delays. In [4], for the system

$$\dot{X}_1(t) = F_1X_1(t) + H_1X_2(t - D_1) + B_1U(t) \tag{1}$$

$$\dot{X}_2(t) = F_2X_2(t) + B_2U(t), \tag{2}$$

a predictor-based controller is designed as

$$U(t) = K_1D_1\int_0^1 \mathrm{e}^{-F_1D_1\theta}H_1X_2\left(t + D_1(\theta - 1)\right)\mathrm{d}\theta + K_1X_1(t) + K_2X_2(t). \tag{3}$$

The above controller is based on a transformation that reduces the system to an equivalent system without state delay. This transformation is

$$Z_1(t) = X_1(t) + D_1\int_0^1 \mathrm{e}^{-F_1D_1\theta}H_1X_2\left(t + D_1(\theta - 1)\right)\mathrm{d}\theta \tag{4}$$

$$Z_2(t) = X_2(t), \tag{5}$$

and it transforms the system (1)–(2) to

$$\dot{Z}_1(t) = F_1Z_1(t) + \mathrm{e}^{-F_1D_1}H_1Z_2(t) + B_1U(t) \tag{6}$$

$$\dot{Z}_2(t) = F_2Z_2(t) + B_2U(t). \tag{7}$$

The importance of the previous transformation, besides transforming the original system to an equivalent one without state delay, is that the system can be linearly parameterized in the state delay, which is the key for designing an adaptive control law.

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Then, assuming that the pair $\left[\begin{pmatrix} F_1 & e^{-D_1F_1}H_1 \\ 0 & F_2 \end{pmatrix}, \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}\right]$ is completely controllable, a state feedback controller $U(t) = K_1Z_1(t) + K_2Z_2(t)$ is designed such that the transformed system is asymptotically stable. It can be shown that controllability of the original system is equivalent to controllability of the transformed system [22]. This design can be also applied in the case where there is a delay in the input, say D_2. In this case after employing the state transformation a predictor feedback is needed for the transformed system. In this case the controller that compensates for D_2 is given by

$$U(t) = Ke^{AD_2}Z(t) + KD_2\int_0^1 e^{AD_2(1-\theta)}BU\left(t - D_2(\theta - 1)\right)d\theta, \quad (8)$$

where

$$A = \begin{bmatrix} F_1 & e^{-D_1F_1}H_1 \\ 0 & F_2 \end{bmatrix} \quad (9)$$

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \quad (10)$$

$$K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}. \quad (11)$$

The controller (8) is the basis of our adaptive design.

2. Problem formulation

In this paper we consider both the state and input delays to be unknown, that is, we consider the system

$$\dot{X}_1(t) = F_1X_1(t) + H_1X_2(t - D_1) + B_1U(t - D_2) \quad (12)$$

$$\dot{X}_2(t) = F_2X_2(t) + B_2U(t - D_2), \quad (13)$$

with D_1 and D_2 unknown. Since D_1 and D_2 are unknown, in addition to the predictor based controller, we must design two estimators, one for each of the delays. We employ projector operators and assume a bound on the length of the delays to be known.

Assumption 1. There exist known constants $\underline{D}_1$, $\overline{D}_1$ and $\overline{D}_2$ such that $D_1 \in [\underline{D}_1, \overline{D}_1]$ and $D_2 \in [0, \overline{D}_2]$.

Our controller is based on the transformed system (i.e., on the system without state delay). As indicated in Section 1, the pair $\left[\begin{pmatrix} F_1 & e^{-D_1F_1}H_1 \\ 0 & F_2 \end{pmatrix}, \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}\right]$ must be completely controllable. Under this assumption we can find a stabilizing state feedback. In the case of unknown state delay D_1, we must assume that there exists a stabilizing state feedback for all values of the state delay in a given interval. We thus make the following assumption.

Assumption 2. The pair $\left[\begin{pmatrix} F_1 & e^{-D_1F_1}H_1 \\ 0 & F_2 \end{pmatrix}, \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}\right]$ is completely controllable $\forall D_1 \in [\underline{D}_1, \overline{D}_1]$. Furthermore, we assume that there exists a triple of vector/matrix-valued functions $(K(D_1), P(D_1), Q(D_1))$ such that $K(D_1) \in C^1\left([\underline{D}_1, \overline{D}_1]\right)$, $P(D_1) \in C^1\left([\underline{D}_1, \overline{D}_1]\right)$, $Q(D_1) \in C^0\left([\underline{D}_1, \overline{D}_1]\right)$, the matrices $P(D_1)$ and $Q(D_1)$ are positive definite and symmetric, and the following Lyapunov equation is satisfied $\forall D_1 \in [\underline{D}_1, \overline{D}_1]$:

$$\begin{aligned}&\left(A(D_1) + BK(D_1)\right)^T P(D_1) + P(D_1)\left(A(D_1) + BK(D_1)\right) \\ &\quad = -Q(D_1), \forall D_1 \in [\underline{D}_1, \overline{D}_1]. \quad (14)\end{aligned}$$

Our final assumption is needed in the choice of the normalization coefficients in the adaptation laws for the delay estimates.

Assumption 3. The quantities $\underline{\lambda} = \inf_{D_1 \in [\underline{D}_1, \overline{D}_1]} \min\{\lambda_{\min}(Q(D_1)), \lambda_{\min}(P(D_1))\}$ and $\overline{\lambda} = \sup_{D_1 \in [\underline{D}_1, \overline{D}_1]} \lambda_{\max}(P(D_1))$ exist and are known.

3. Controller design

We first rewrite (12)–(13) using a PDE representation of the delayed states and control as

$$\dot{X}_1(t) = F_1X_1(t) + H_1\xi(0, t) + B_1u(0, t) \quad (15)$$

$$D_1\xi_t(x, t) = \xi_x(x, t) \quad (16)$$

$$\xi(1, t) = X_2(t) \quad (17)$$

$$\dot{X}_2(t) = F_2X_2(t) + B_2u(0, t) \quad (18)$$

$$D_2u_t(x, t) = u_x(x, t) \quad (19)$$

$$u(1, t) = U(t), \quad (20)$$

where $x \in [0, 1]$. We assume that the infinite-dimensional states $\xi(x, t), u(x, t), x \in [0, 1]$ are available for measurement. This assumption is not in contradiction with the assumption that the convection speeds $1/D_1$ and $1/D_2$ are unknown. As restrictive as the requirement for measurement of $\xi(x, t), u(x, t), x \in [0, 1]$ may appear, we do not believe that the delay-adaptive problem without such measurements is solvable globally because it cannot be formulated as linearly parameterized in the unknown delays D_1 and D_2.

The transport PDE states can be expressed in terms of the past values of X_2 and U as

$$\xi(x, t) = X_2(t + D_1(x - 1)) \quad (21)$$

$$u(x, t) = U(t + D_2(x - 1)). \quad (22)$$

Using the certainty equivalence principle the controller (8) is taken as

$$\begin{aligned}U(t) = {}& K(\hat{D}_1)e^{A(\hat{D}_1)\hat{D}_2(t)}\begin{bmatrix} X_1(t) + \hat{D}_1(t)\int_0^1 e^{-F_1\hat{D}_1(t)y}H_1\xi(y, t)dy \\ X_2(t) \end{bmatrix} \\ &+ K(\hat{D}_1)\hat{D}_2(t)\int_0^1 e^{A(\hat{D}_1)\hat{D}_2(t)(1-y)}Bu(y, t)dy. \quad (23)\end{aligned}$$

The update laws for the estimations of the unknown delays D_1 and D_2 are given by

$$\dot{\hat{D}}_1(t) = \gamma_1\mathrm{Proj}_{[\underline{D}_1, \overline{D}_1]}\{\tau_{D_1}\} \quad (24)$$

$$\dot{\hat{D}}_2(t) = \gamma_2\mathrm{Proj}_{[0, \overline{D}_2]}\{\tau_{D_2}\}, \quad (25)$$

where the projector operators are defined as

$$\mathrm{Proj}_{[\underline{D}_i, \overline{D}_i]}\{\tau_{D_i}\} = \begin{cases} 0 & \text{if } \hat{D}_i = \underline{D}_i \text{ and } \tau_{D_i} < 0 \\ 0 & \text{if } \hat{D}_i = \overline{D}_i \text{ and } \tau_{D_i} > 0 \\ \tau_{D_i} & \text{else} \end{cases}, \quad (26)$$

and where

$$\tau_{D_1} = \frac{\int_0^1 (1+x)w(x, t)K(\hat{D}_1)e^{A(\hat{D}_1)\hat{D}_2(t)x}dx - \frac{2}{a_2}Z^T(t)P(\hat{D}_1)}{\Gamma(t)} \times R_2(t) \quad (27)$$

$$\tau_{D_2} = -\frac{\int_0^1 (1+x)w(x, t)K(\hat{D}_1)e^{A(\hat{D}_1)\hat{D}_2(t)x}dx}{\Gamma(t)} \times \left(Bu(0, t) + A(\hat{D}_1)Z(t)\right) \quad (28)$$

$$\begin{aligned}\Gamma(t) = {}& 1 + Z^T(t)P(\hat{D}_1)Z(t) + a_2\int_0^1 (1+x)w^2(x, t)dx \\ &+ k\int_0^1 (1+x)\xi^T(x, t)\xi(x, t)dx, \quad (29)\end{aligned}$$

with

$$k \le \frac{\lambda D_1}{8} \tag{30}$$

$$a_2 \ge \frac{\overline{D}_2 \sup_{\hat{D}_1 \in [\underline{D}_1, \overline{D}_1]} |P(\hat{D}_1)B|^2}{\underline{\lambda}}. \tag{31}$$

In the above relation we use the following signals which are derived in the stability analysis of the closed-loop system

$$Z_1(t) = X_1(t) + \hat{D}_1(t) \int_0^1 e^{-\hat{D}_1(t)F_1 y} H_1 \xi(y,t) dy \tag{32}$$

$$Z_2(t) = X_2(t) \tag{33}$$

$$R_2(t) = \begin{bmatrix} e^{-\hat{D}_1(t)F_1} H_1 Z_2(t) - H_1\xi(0,t) + \hat{D}_1(t) F_1 \int_0^1 e^{-\hat{D}_1(t)F_1 y} H_1 \xi(y,t) dy \\ 0 \end{bmatrix} \tag{34}$$

and the transformed infinite dimensional state of the actuator

$$w(x,t) = u(x,t) - K(\hat{D}_1)\Big(e^{A(\hat{D}_1)\hat{D}_2(t)x} Z(t) + \hat{D}_2(t) \times \int_0^x e^{A(\hat{D}_1)\hat{D}_2(t)(x-y)} Bu(y,t) dy \Big). \tag{35}$$

4. Stability analysis

This section is devoted to the proof of the main result. We start by giving the main theorem and in the rest of the section we prove it using a series of technical lemmas.

Theorem 1. *Let Assumptions 1–3 hold. Then system (12)–(13) with the controller (23) and the update laws (24)–(25) is stable in the sense that there exist constants R and ρ such that*

$$\Omega(t) \le R\left(e^{\rho\Omega(0)} - 1\right), \tag{36}$$

where

$$\Omega(t) = |X(t)|^2 + \|\xi(t)\|^2 + \|u(t)\|^2 + \tilde{D}_1^2(t) + \tilde{D}_2^2(t), \tag{37}$$

and

$$\|\xi(t)\|^2 = \int_0^1 \xi(y,t)^T \xi(y,t) dy = \frac{1}{D_1}\int_{t-D_1}^t X_2(\theta)^T X_2(\theta) d\theta \tag{38}$$

$$\|u(t)\|^2 = \int_0^1 u^2(y,t) dy = \frac{1}{D_2}\int_{t-D_2}^t U^2(\theta) d\theta. \tag{39}$$

Furthermore

$$\lim_{t\to\infty} X(t) = 0 \tag{40}$$

$$\lim_{t\to\infty} U(t) = 0. \tag{41}$$

We start proving the above theorem by first transforming the system (15)–(20) using the transformations (32)–(33) and (35). By differentiating with respect to time (32) and (33) and by using (15) and (18) we get

$$\begin{aligned} \dot{Z}_1(t) &= F_1 X_1(t) + H_1\xi(0,t) + B_1 u(0,t) \\ &\quad + \dot{\hat{D}}_1(t) \int_0^1 e^{-\hat{D}_1(t)F_1 y} H_1 \xi(y,t) dy \\ &\quad - \hat{D}_1(t)\dot{\hat{D}}_1(t) \int_0^1 F_1 y e^{-\hat{D}_1(t)F_1 y} H_1 \xi(y,t) dy \\ &\quad + \hat{D}_1(t) \int_0^1 e^{-\hat{D}_1(t)F_1 y} H_1 \xi_t(y,t) dy \end{aligned} \tag{42}$$

$$\dot{Z}_2(t) = F_2 X_2(t) + B_2 u(0,t). \tag{43}$$

Using relations (16) and (17), the fact that $\frac{\hat{D}_1}{D_1} = 1 - \frac{\tilde{D}_1}{D_1}$ and integrating by parts the last integral in (42) we obtain

$$\begin{aligned} \dot{Z}_1(t) &= F_1 X_1(t) + H_1\xi(0,t) + B_1 u(0,t) \\ &\quad + \dot{\hat{D}}_1(t) \int_0^1 e^{-\hat{D}_1(t)F_1 y} H_1 \xi(y,t) dy \\ &\quad - \hat{D}_1(t)\dot{\hat{D}}_1(t) \int_0^1 F_1 y e^{-\hat{D}_1(t)F_1 y} H_1 \xi(y,t) dy \\ &\quad + \left(1 - \frac{\tilde{D}_1}{D_1}\right)\left(e^{-\hat{D}_1(t)F_1} H_1 X_2(t) - H_1\xi(0,t)\right) \\ &\quad + \left(1 - \frac{\tilde{D}_1}{D_1}\right) \int_0^1 \hat{D}_1(t) F_1 e^{-\hat{D}_1(t)F_1 y} H_1 \xi(y,t) dy \end{aligned} \tag{44}$$

$$\dot{Z}_2(t) = F_2 X_2(t) + B_2 u(0,t). \tag{45}$$

Using (32)–(33) and after some algebra we arrive at

$$\begin{aligned} \dot{Z}(t) &= \begin{bmatrix} \dot{Z}_1(t) \\ \dot{Z}_2(t) \end{bmatrix} \\ &= A(\hat{D}_1) Z(t) + Bu(0,t) + \dot{\hat{D}}_1(t) R_1(t) - \frac{\tilde{D}_1}{D_1} R_2(t), \end{aligned} \tag{46}$$

where $R_2(t)$ is defined in (34) and

$$A(\hat{D}_1) = \begin{bmatrix} F_1 & e^{-\hat{D}_1(t)F_1} H_1 \\ 0 & F_2 \end{bmatrix} \tag{47}$$

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{48}$$

$$R_1(t) = \begin{bmatrix} \int_0^1 \left(I - \hat{D}_1(t) F_1 y\right) e^{-\hat{D}_1(t)F_1 y} H_1 \xi(y,t) dy \\ 0 \end{bmatrix}. \tag{49}$$

Using relation (35) for $x = 0$ we get that

$$\dot{Z}(t) = \left(A(\hat{D}_1) + BK(\hat{D}_1)\right) Z(t) + Bw(0,t) + \dot{\hat{D}}_1(t) R_1(t) - \frac{\tilde{D}_1}{D_1} R_2(t). \tag{50}$$

Moreover the transformation of the actuator state w satisfies

$$D_2 w_t(x,t) = w_x(x,t) + \frac{\tilde{D}_1}{D_1} D_2 p_1(x,t) - \tilde{D}_2 p_2(x,t) - D_2 \dot{\hat{D}}_1(t) q_1(x,t) - D_2 \dot{\hat{D}}_2(t) q_2(x,t), \tag{51}$$

where

$$p_1(x,t) = K(\hat{D}_1) e^{A(\hat{D}_1)\hat{D}_2(t)x} R_2(t) \tag{52}$$

$$\begin{aligned} q_1(x,t) &= \int_0^x \left(\frac{\partial K(\hat{D}_1)}{\partial \hat{D}_1} + K(\hat{D}_1) \frac{\partial A(\hat{D}_1)}{\partial \hat{D}_1}(x-y) \right) \\ &\quad \times \hat{D}_2(t) e^{\hat{D}_2(t) A(\hat{D}_1)(x-y)} Bu(y,t) dy \\ &\quad + \left(\frac{\partial K(\hat{D}_1)}{\partial \hat{D}_1} + K(\hat{D}_1) \frac{\partial A(\hat{D}_1)}{\partial \hat{D}_1} \hat{D}_2(t) x \right) \\ &\quad \times e^{\hat{D}_2(t) A(\hat{D}_1)x} Z(t) + K(\hat{D}_1) e^{\hat{D}_2(t) A(\hat{D}_1)x} R_1(t) \end{aligned} \tag{53}$$

$$\begin{aligned} p_2(x,t) &= K(\hat{D}_1) e^{\hat{D}_2(t) A(\hat{D}_1)x} Bu(0,t) + K(\hat{D}_1) A(\hat{D}_1) \\ &\quad \times e^{\hat{D}_2(t) A(\hat{D}_1)x} Z(t) \end{aligned} \tag{54}$$

$$\begin{aligned} q_2(x,t) &= K(\hat{D}_1) \int_0^x \left(I + \hat{D}_2(t)(x-y) A(\hat{D}_1)\right) e^{A(\hat{D}_1)\hat{D}_2(t)(x-y)} \\ &\quad \times Bu(y,t) dy. \end{aligned} \tag{55}$$

Thus now, system (15)–(20) is mapped to the target system that is comprised of (50) and (51). Moreover, the inverse transformation of the state $X(t)$ is easily obtained from Eqs. (32)–(33) and the inverse transformation of (35) is given by

$$u(x,t) = w(x,t) + K(\hat{D}_1)\Bigg(\mathrm{e}^{\left(A(\hat{D}_1)+BK(\hat{D}_1)\right)\hat{D}_2(t)x}Z(t) + \hat{D}_2(t)\int_0^x \mathrm{e}^{\left(A(\hat{D}_1)+BK(\hat{D}_1)\right)\hat{D}_2(t)(x-y)}Bw(y,t)\mathrm{d}y\Bigg). \tag{56}$$

We first prove that the signals in (50) and (51) that multiply the "disturbances" $\dot{\hat{D}}_i$ and $\tilde{D}_i$, $i = 1, 2.$, are bounded with respect to the system's transformed states $Z(t)$, $\xi(x,t)$ and $w(x,t)$. Before doing that, we point out that boundness of the transformed states is equivalent to boundness of the original states.

Lemma 1. *There exist constants M_u, M_w, M_X and M_Z such that*

$$\|u(t)\|^2 \le M_u\left(\|w(t)\|^2 + |Z(t)|^2\right) \tag{57}$$

$$|X(t)|^2 \le M_X\left(|Z(t)|^2 + \|\xi(t)\|^2\right) \tag{58}$$

$$\|w(t)\|^2 \le M_w\left(\|u(t)\|^2 + |Z(t)|^2\right) \tag{59}$$

$$|Z(t)|^2 \le M_Z\left(|X(t)|^2 + \|\xi(t)\|^2\right). \tag{60}$$

Proof. First observe that the signals $K(\hat{D}_1)$, $P(\hat{D}_1)$ and $A(\hat{D}_1)$ are continuously differentiable with respect to $\hat{D}_1$. Moreover, since $\hat{D}_1$ and $\hat{D}_2$ are uniformly bounded, the signals $K(\hat{D}_1)$, $P(\hat{D}_1)$ and $A(\hat{D}_1)$ and their derivatives are also uniformly bounded. Denote by M_K, M_P and M_A the bounds of $K(\hat{D}_1)$, $P(\hat{D}_1)$ and $A(\hat{D}_1)$ respectively, and with M_K', M_P' and M_A' the bounds of their derivatives. From relations (32)–(33) and (35), (56) and using Young's and Cauchy–Schwartz's inequalities it easy to show that the above bounds hold with

$$M_u = 3\max\left\{1 + M_K^2\overline{D}_2^2\mathrm{e}^{2\overline{D}_2(M_A+|B|M_K)}|B|^2,\ M_K^2\mathrm{e}^{2\overline{D}_2(M_A+|B|M_K)}\right\} \tag{61}$$

$$M_Z = 2\max\left\{1,\ \overline{D}_1^2\mathrm{e}^{2|F_1|\overline{D}_1}|H_1|^2\right\} \tag{62}$$

$$M_X = 2\max\left\{1,\ \overline{D}_1^2\mathrm{e}^{2|F_1|\overline{D}_1}|H_1|^2\right\} \tag{63}$$

$$M_w = 3\max\left\{1 + M_K^2\overline{D}_2^2\mathrm{e}^{2M_A\overline{D}_2}|B|^2,\ M_K^2\mathrm{e}^{2M_A\overline{D}_2}\right\}. \quad \square \tag{64}$$

We are now ready to state the following lemma

Lemma 2. *There exist constants M_{R_1}, M_{R_2}, M_{p_1}, M_{p_2}, M_{q_1} and M_{q_2} such that the following bounds hold*

$$|R_1(t)|^2 \le M_{R_1}\|\xi(t)\|^2 \tag{65}$$

$$|R_2(t)|^2 \le M_{R_2}\left(|Z(t)|^2 + |\xi(0,t)|^2 + \|\xi(t)\|^2\right) \tag{66}$$

$$p_1^2(x,t) \le M_{p_1}\left(|Z(t)|^2 + |\xi(0,t)|^2 + \|\xi(t)\|^2\right) \tag{67}$$

$$p_2^2(x,t) \le M_{p_2}\left(|Z(t)|^2 + u^2(0,t)\right) \tag{68}$$

$$q_1^2(x,t) \le M_{q_1}\left(|Z(t)|^2 + \|u(t)\|^2 + \|\xi(t)\|^2\right) \tag{69}$$

$$q_2^2(x,t) \le M_{q_2}\|u(t)\|^2, \tag{70}$$

for all $x \in [0, 1]$.

Proof. From relations (49) and (34) and by using Young's and Cauchy–Schwartz's inequalities we get the bounds for $R_1(t)$ and $R_2(t)$ with

$$M_{R_1} = \left(1 + \overline{D}_1|F_1|\right)^2\mathrm{e}^{2\overline{D}_2|F_1|}|H_1|^2 \tag{71}$$

$$M_{R_2} = 3|H_1|^2\max\left\{\mathrm{e}^{2\overline{D}_2|F_1|}|F_1|^2,\ 1,\ \overline{D}_1^2|F_1|^2\mathrm{e}^{2\overline{D}_2|F_1|}\right\}. \tag{72}$$

Using relations (52)–(55) together with Young's and Cauchy–Schwartz's inequalities, relations (65)–(66) and (57) we get the bounds of the lemma with

$$M_{p_1} = M_K^2\mathrm{e}^{2M_A\overline{D}_2}M_{R_2} \tag{73}$$

$$M_{p_2} = 3\max\left\{M_K^2\mathrm{e}^{2M_A\overline{D}_2}|B|^2,\ M_K^2M_A^2\mathrm{e}^{2M_A\overline{D}_2}\right\} \tag{74}$$

$$M_{q_1} = 3\max\Big\{\left(M_K'\overline{D}_2 + \overline{D}_2M_KM_A'\right)^2|B|^2\mathrm{e}^{2M_A\overline{D}_2},\ M_K^2\mathrm{e}^{2M_A\overline{D}_2}M_{R_1},\ \left(M_K' + M_KM_A'\overline{D}_2\right)^2\mathrm{e}^{2M_A\overline{D}_2}\Big\} \tag{75}$$

$$M_{q_2} = M_K^2\left(1 + \overline{D}_2M_A\right)^2\mathrm{e}^{2M_A\overline{D}_2}. \quad \square \tag{76}$$

Lemma 3. *There exist constants k, a_2, γ_1 and γ_2 such that for the Lyapunov function*

$$V(t) = D_2\log\left(1 + \Xi(t)\right) + a_2D_2\frac{\tilde{D}_1^2(t)}{D_1\gamma_1} + a_2\frac{\tilde{D}_2^2(t)}{\gamma_2}, \tag{77}$$

where

$$\Xi(t) = Z(t)^TP\left(\hat{D}_1\right)Z(t) + k\int_0^1(1+x)\xi^T(x,t)\xi(x,t)\mathrm{d}x + a_2\int_0^1(1+x)w^2(x,t)\mathrm{d}x, \tag{78}$$

the following holds

$$V(t) \le V(0). \tag{79}$$

Proof. Taking the time derivative of the above function we obtain

$$\begin{aligned}\dot{V}(t) = {} & -2a_2D_2\frac{\tilde{D}_1}{D_1\gamma_1}\left(\dot{\hat{D}}_1(t) - \gamma_1\tau_{D_1}\right) - 2a_2\frac{\tilde{D}_2}{\gamma_2}\left(\dot{\hat{D}}_2(t) - \gamma_2\tau_{D_2}\right)\\ & + \frac{D_2}{1+\Xi(t)}\Bigg(-Z^T(t)Q(\hat{D}_1)Z(t) + 2Z^T(t)P(\hat{D}_1)Bw(0,t)\\ & + \frac{2k}{D_1}Z_2^T(t)Z_2(t) - \frac{k}{D_1}|\xi(0,t)|^2\\ & - \frac{a_2}{D_2}w^2(0,t) - \frac{k}{D_1}\int_0^1\xi^T(x,t)\xi(x,t)\mathrm{d}x\\ & - \frac{a_2}{D_2}\int_0^1 w^2(x,t)\mathrm{d}x + \dot{\hat{D}}_1(t)\Bigg(Z^T(t)\frac{\partial P(\hat{D}_1)}{\partial\hat{D}_1}Z(t)\\ & + 2Z^T(t)P(\hat{D}_1)R_1(t) - 2a_2\int_0^1(1+x)w(x,t)q_1(x,t)\mathrm{d}x\Bigg)\\ & - 2a_2\dot{\hat{D}}_2(t)\int_0^1(1+x)w(x,t)q_2(x,t)\mathrm{d}x\Bigg).\end{aligned} \tag{80}$$

Using the properties of the projector operators and relations (24)–(25) we get

$$\begin{aligned}\dot{V}(t) \le {} & \frac{D_2}{1+\Xi(t)}\Bigg(-Z^T(t)Q(\hat{D}_1)Z(t) + \frac{2k}{D_1}Z_2^T(t)Z_2(t)\\ & - \frac{k}{D_1}|\xi(0,t)|^2 - \frac{a_2}{D_2}w^2(0,t) + Z^T(t)P(\hat{D}_1)Bw(0,t)\\ & - \frac{k}{D_1}\int_0^1\xi^T(x,t)\xi(x,t)\mathrm{d}x - \frac{a_2}{D_2}\int_0^1 w^2(x,t)\mathrm{d}x\\ & + \dot{\hat{D}}_1(t)\Bigg(Z^T(t)\frac{\partial P(\hat{D}_1)}{\partial\hat{D}_1}Z(t) + 2Z^T(t)P(\hat{D}_1)R_1(t)\end{aligned}$$

$$-2a_2\int_0^1(1+x)w(x,t)q_1(x,t)\mathrm{d}x\Big)$$
$$-2a_2\dot{\hat{D}}_2(t)\int_0^1(1+x)w(x,t)q_2(x,t)\mathrm{d}x\Bigg). \quad (81)$$

Then using Young's inequality and (30)–(31) we get

$$\dot{V}(t) \le \frac{D_2}{1+\Xi(t)}\Bigg(-\frac{\underline{\lambda}}{2}|Z(t)|^2-\frac{k}{D_1}|\xi(0,t)|^2-\frac{k}{D_1}\|\xi(t)\|^2 -\frac{a_2}{2D_2}w^2(0,t)-\frac{a_2}{D_2}\|w(t)\|^2+\dot{\hat{D}}_1(t)\Big(Z^T(t)\frac{\partial P(\hat{D}_1)}{\partial\hat{D}_1}Z(t) +2Z^T(t)P(\hat{D}_1)R_1(t)-2a_2\int_0^1(1+x)w(x,t)q_1(x,t)\mathrm{d}x\Big) -2a_2\dot{\hat{D}}_2(t)\int_0^1(1+x)w(x,t)q_2(x,t)\mathrm{d}x\Bigg). \quad (82)$$

Using bounds (65)–(70) together with relations (26) and (24)–(25), and by employing Young's inequality one more time we get

$$|\dot{\hat{D}}_1(t)| \le \gamma_1|\tau_{D_1}| \le \gamma_1 M_1\frac{\left(|Z(t)|^2+|\xi(0,t)|^2+\|\xi(t)\|^2+\|w(t)\|^2\right)}{1+\Xi(t)} \quad (83)$$

$$|\dot{\hat{D}}_2(t)| \le \gamma_2|\tau_{D_2}| \le \gamma_2 M_2\frac{\left(|Z(t)|^2+w^2(0,t)+\|w(t)\|^2\right)}{1+\Xi(t)}, \quad (84)$$

where

$$M_1=\max\left\{M_{p_1}+\frac{1}{a_2}M_P+\frac{1}{a_2}M_PM_{R_2},1\right\} \quad (85)$$

$$M_2=\max\left\{1,2M_{p_2},2M_{p_2}M_K^2+M_{p_2}\right\}. \quad (86)$$

Plugging in the above bound to (82) (and applying once more Young's and Cauchy–Schwartz's inequalities) we get

$$\dot{V}(t) \le \frac{D_2}{1+\Xi(t)}\Bigg(-\frac{\underline{\lambda}}{2}|Z(t)|^2-\frac{k}{D_1}|\xi(0,t)|^2-\frac{k}{D_1}\|\xi(t)\|^2 -\frac{a_2}{2D_2}w^2(0,t)-\frac{a_2}{D_2}\|w(t)\|^2 +B_1\gamma_1\frac{\left(|Z(t)|^2+|\xi(0,t)|^2+\|\xi(t)\|^2+\|w(t)\|^2\right)}{1+\Xi(t)} \times\left(|Z(t)|^2+\|\xi(t)\|^2+\|w(t)\|^2\right) +B_2\gamma_2\frac{\left(|Z(t)|^2+w^2(0,t)+\|w(t)\|^2\right)}{1+\Xi(t)} \times\left(|Z(t)|^2+\|w(t)\|^2\right)\Bigg), \quad (87)$$

where

$$B_1 = M_1\max\left\{M_P'+M_P+a_2M_{q_1}+2a_2M_{q_1}M_u+2a_2M_{q_1}, M_PM_{R_1}+2a_2M_{q_1},2a_2+2a_2M_{q_1}M_u\right\} \quad (88)$$

$$B_2=2M_2a_2\left(1+M_{q_2}M_u\right). \quad (89)$$

Now by defining the constants

$$m_1=\min\left\{\frac{\underline{\lambda}}{2},\frac{k}{\overline{D}_1},\frac{a_2}{2\overline{D}_2}\right\} \quad (90)$$

$$m_2=\frac{\max\{B_1,B_2\}}{\min\{\underline{\lambda},k,a_2\}}, \quad (91)$$

we get

$$\dot{V}(t) \le -\frac{D_2}{1+\Xi(t)}\left(m_1-m_2(\gamma_1+\gamma_2)\right)\Big(|Z(t)|^2+w^2(0,t) +\|w(t)\|^2+|\xi(0,t)|^2+\|\xi(t)\|^2\Big). \quad (92)$$

Thus when $\gamma_1+\gamma_2\le\frac{m_1}{m_2}$, $\dot{V}(t)$ is negative definite and thus

$$V(t)\le V(0). \quad \square \quad (93)$$

To prove the stability bound of Theorem 1 we use the following lemma.

Lemma 4. *There exist constants $\underline{M}$ and $\overline{M}$ such that*

$$\underline{M}\,\Xi(t)\le\Pi(t)\le\overline{M}\,\Xi(t), \quad (94)$$

where

$$\Pi(t)=|X(t)|^2+\|\xi(t)\|^2+\|u(t)\|^2. \quad (95)$$

Proof. Immediate, using (57)–(60) with

$$\overline{M}=\max\{M_u+M_X,M_X+1\} \quad (96)$$

$$\underline{M}=\max\{M_w+M_Z,M_Z+1\}. \quad \square \quad (97)$$

We are now ready to derive the stability estimate of Theorem 1. Using (77) it follows that

$$\Xi(t)\le\left(\mathrm{e}^{\frac{V(t)}{\underline{D}_2}}-1\right) \quad (98)$$

$$\tilde{D}_1^2+\tilde{D}_2^2\le C_2\frac{V(t)}{\underline{D}_2}\le C_2\left(\mathrm{e}^{\frac{V(t)}{\underline{D}_2}}-1\right), \quad (99)$$

where

$$C_2=\frac{\left(\gamma_2\overline{D}_2+\gamma_1\overline{D}_1\right)}{a_2}. \quad (100)$$

Consequently

$$\Omega(t)\le\left(\overline{M}+C_2\right)\left(\mathrm{e}^{\frac{V(t)}{\underline{D}_2}}-1\right). \quad (101)$$

Moreover, from (77) we take

$$V(0) \le \max\left\{\overline{\lambda},k,a_2\right\}\left(|Z(0)|^2+\|\xi(0)\|^2+\|w(0)\|^2\right) +\max\left\{\frac{a_2}{\gamma_2},\frac{a_2\overline{D}_2}{\gamma_1\underline{D}_1}\right\}\left(\tilde{D}_1^2(0)+\tilde{D}_2^2(0)\right), \quad (102)$$

and using Lemma 4 we have

$$V(0)\le C_3\Omega(0), \quad (103)$$

where

$$C_3=\max\left\{\max\left\{\frac{1}{2\gamma_2},\frac{\overline{D}_2}{2\gamma_1\underline{D}_1}\right\},\frac{\max\left\{\overline{\lambda},k,a_2\right\}}{\underline{M}}\right\}. \quad (104)$$

Thus, by setting

$$R=\overline{M}+C_2 \quad (105)$$

$$\rho=C_3, \quad (106)$$

we get the stability result in Theorem 1.

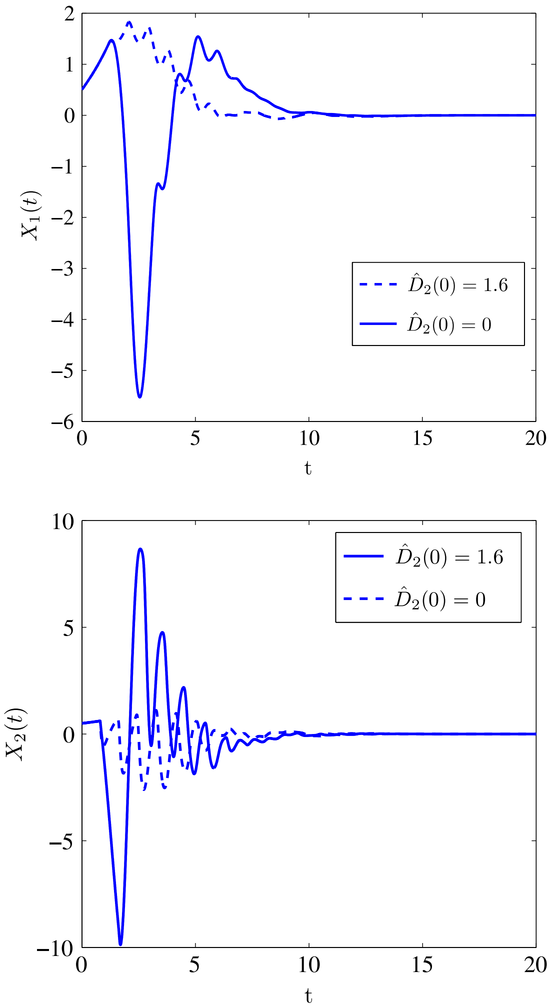

Fig. 1. System's response for the cases $\hat{D}_2(0) = 0$ and $\hat{D}_2(0) = 1.6$.

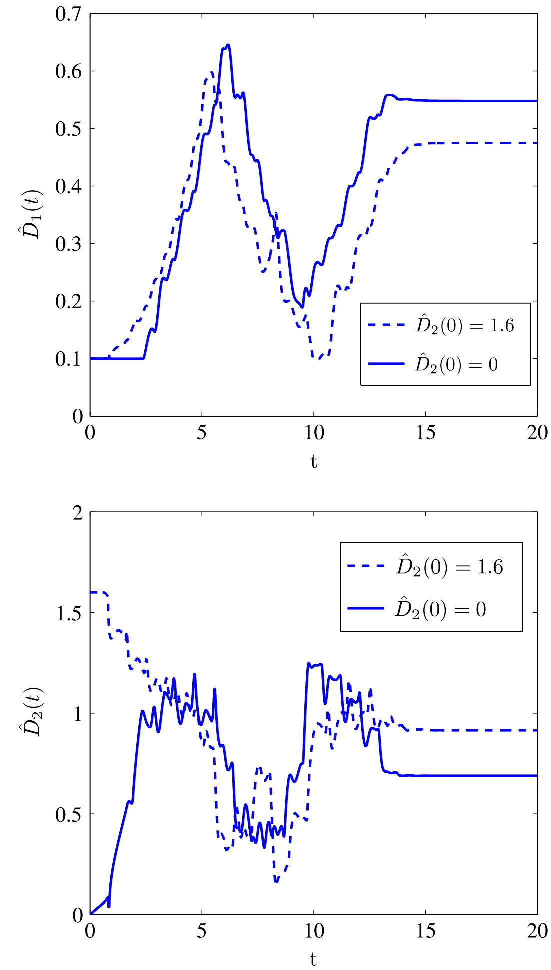

Fig. 2. The estimations of the unknown delays for the cases $\hat{D}_2(0) = 0$ and $\hat{D}_2(0) = 1.6$.

We now turn our attention to proving the convergence of $X(t)$ and $U(t)$ to zero. We use here an alternative to Barbalat's Lemma from [23]. We first point out that from (93) it follows that $|Z(t)|$, $\|w(t)\|$, $\|\xi(t)\|$, $\hat{D}_1$ and $\hat{D}_2$ are uniformly bounded. Moreover, using (57) and (58) we get the uniform boundness of $|X(t)|$ and $\|u(t)\|$. Using (23) it follows that $U(t)$ is uniformly bounded. From (24)–(25), (83)–(84) and (50) we conclude that $\frac{\mathrm{d}X^2(t)}{\mathrm{d}t}$ is uniformly bounded. Finally, since from (92) it turns out that $|Z(t)|$ and $\|\xi(t)\|$ are square integrable, using (58) and the alternative to Barbalat's Lemma from [23], we conclude that $\lim_{t\to\infty} X(t) = 0$. We now turn our attention to proving the convergence of $U(t)$. Using (92) we also get that $\|w(t)\|$ is square integrable in time. Thus, with the help of (57) and by the square integrability of $|Z(t)|$ we conclude using (23) that $U(t)$ is square integrable. It only remains to show that $\frac{\mathrm{d}U^2(t)}{\mathrm{d}t}$ is uniformly bounded. Hence, it is sufficient to show that $\dot{U}(t)$ is uniformly bounded. From (23) one can observe that since $\dot{\hat{D}}_1$ and $\dot{\hat{D}}_2$ are uniformly bounded, with the help of (16) and (19) we conclude the uniform boundness of $\frac{\mathrm{d}U^2(t)}{\mathrm{d}t}$.

5. Simulations

We give here a simulation example to illustrate the effectiveness of the proposed adaptive scheme. We choose a second order feedforward system with parameters $F_1 = F_2 = 0.25$, $H_1 = 1$, $D_1 = 0.4$ and $D_2 = 0.8$. This is an unstable system with two poles at 0.25. The lower bounds for the unknown delays are $\underline{D}_1 = 0.1$ for D_1 and 0 for D_2. Analogously the upper bounds are chosen as twice the real values of the delays i.e., $\overline{D}_1 = 0.8$ and $\overline{D}_2 = 1.6$. The initial conditions are chosen as $X_1(0) = 0.5$, $X_2(0) = 0.5$ and $X_2(\theta) = 0.5$, $\forall\theta \in [-D_1, 0]$, and finally $\hat{D}_1(0) = \underline{D}_1 = 0.1$. The controller parameters are chosen as $a_2 = 200$, $k = 0.005$, $\gamma_1 = 25$, $\gamma_2 = 25$ and $K(\hat{D}_1) = \left[-10.0625\mathrm{e}^{0.25\hat{D}_1} \quad -8.5\right]$.

Figs. 1–3 show two distinct simulations, starting from two extreme initial values for the input delay estimate, one at zero, and the other at twice the true delay value. In Figs. 1 and 3 we observe that, as Theorem 1 predicts, convergence to zero is achieved for the states and the input, despite starting with initial estimate for the input delay at the two extreme values and for the state delay at the lower bound. In Fig. 2 one can see the evolution of the estimates for the two distinct simulation cases. The estimates for the two delays, after a transient response, converge to stabilizing for the system values.

6. Conclusions

In this paper we presented an adaptive control design for a system in feedforward form with simultaneous unknown input and state delays. The design of the controller is based on predictor feedback. The update laws for the estimation of the unknown delays are based on the construction of a Lyapunov function with

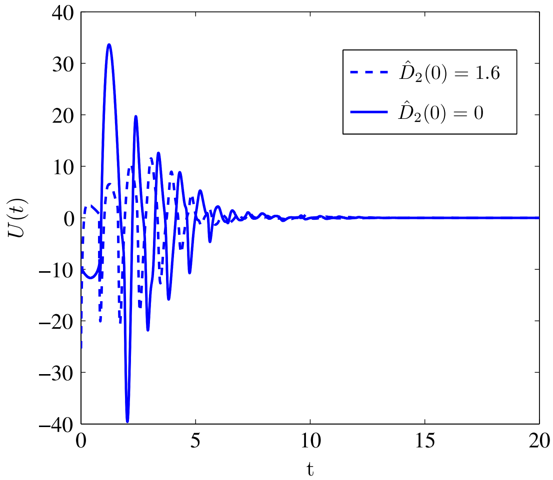

Fig. 3. The control effort for the cases $\hat{D}_2(0) = 0$ and $\hat{D}_2(0) = 1.6$.

normalization. Convergence to zero is then proved using the linear boundness of the relative signals.

References

[1] Z. Artstein, Linear systems with delayed controls: a reduction, IEEE Transactions on Automatic Control 27 (1982) 869–879.
[2] Y.A. Fiagbedzi, A.E. Pearson, Feedback stabilization of linear autonomous time lag systems, IEEE Transactions on Automatic Control 31 (1986) 847–855.
[3] M. Jankovic, Forwarding, backstepping, and finite spectrum assignment for time delay systems, in: 2006 American Control Conference, Minneapolis, Minnesota, 2006.
[4] M. Jankovic, Recursive predictor design for linear systems with time delay, in: 2008 American Control Conference, Seattle, Washington, 2008.
[5] M. Krstic, On compensating long actuator delays in nonlinear control, IEEE Transactions on Automatic Control 53 (2008) 1684–1688.
[6] M. Krstic, Input delay compensation for forward complete and feedforward nonlinear systems, IEEE Transactions on Automatic Control 55 (2010) 287–303.
[7] W.H. Kwon, A.E. Pearson, Feedback stabilization of linear systems with delayed control, IEEE Transactions on Automatic Control 25 (1980) 266–269.
[8] S. Mondie, W. Michiels, Finite spectrum assignment of unstable time-delay systems with a safe implementation, IEEE Transactions on Automatic Control 48 (2003) 2207–2212.
[9] A.W. Olbrot, Stabilizability, detectability, and spectrum assignment for linear autonomous systems with general time delays, IEEE Transactions on Automatic Control 23 (1978) 887–890.
[10] N. Bekiaris-Liberis, M. Krstic, On stabilizing strict-feedback linear systems with delayed integrators, in: 2009 Mediterranean Conference on Control and Automation, Thessaloniki, Greece, 2009.
[11] J.J. Loiseau, Algebraic tools for the control and stabilization of time-delay systems, Annual Reviews in Control 24 (2000) 135–149.
[12] A.Z. Manitius, A.W. Olbrot, Finite spectrum assignment for systems with delays, IEEE Transactions on Automatic Control 24 (1979) 541–553.
[13] K. Watanabe, E. Nobuyama, T. Kitamori, M. Ito, A New Algorithm for Finite Spectrum Assignment of Single-Input Systems with Time Delay, IEEE Transactions on Automatic Control 37 (1992) 1377–1383.
[14] S. Diop, I. Kolmanovsky, P.E. Moraal, M. van Nieuwstadt, Preserving stability/performance when facing an unknown time-delay, Control Engineering Practice 9 (2001) 1319–1325.
[15] M. Krstic, A. Banaszuk, Multivariable adaptive control of instabilities arising in jet engines, Control Engineering Practice 14 (2006) 833–842.
[16] J-P. Richard, Time-delay systems: an overview of some recent advances and open problems, Automatica 39 (2003) 1667–1694.
[17] S. Evesque, A.M. Annaswamy, S. Niculescu, A.P. Dowling, Adaptive control of a class of time-delay systems, ASME Transactions on Dynamics, Systems, Measurement, and Control 125 (2003) 186–193.
[18] S.-I. Niculescu, A.M. Annaswamy, An adaptive Smith- controller for time-delay systems with relative degree $n \geq 2$, Systems and Control Letters 49 (2003) 347–358.
[19] D. Bresch-Pietri, Miroslav Krstic, Delay-adaptive full-state predictor feedback for systems with unknown long actuator delay, in: 2009 American Control Conference, St. Louis, Missouri, 2009.
[20] D. Bresch-Pietri, Miroslav Krstic, Adaptive trajectory tracking despite unknown input delay and plant parameters, Automatica 45 (2009) 2074–2081.
[21] M. Krstic, A. Smyshlyaev, Adaptive boundary control for unstable parabolic PDEs—Part I: Lyapunov design, IEEE Transactions on Automatic Control 53 (2008) 1575–1591.
[22] K.P. Bhat, H.N. Koivo, Model characterization of controllability and observability in time-delay systems, IEEE Transactions on Automatic Control 21 (2) (1976) 292–293.
[23] W.-J. Liu, M. Krstic, Adaptive control of Burgers' equation with unknown viscosity, International Journal of Adaptive Control and Signal Processing 15 (2001) 745–766.
[24] K. Gu, S.-I. Niculescu, Survey on recent results in the stability and control of time-delay systems, ASME Transactions on Dynamics, Systems, Measurement, and Control 125 (2003) 158–165.
[25] M. Jankovic, Control Lyapunov–Razumikhin functions and robust stabilization of time delay systems, IEEE Transactions on Automatic Control 46 (2001) 1048–1060.
[26] M. Jankovic, Control of nonlinear systems with time delay, in: 2003 IEEE Conference on Decision and Control, Maui, Hawai, 2003.
[27] M. Jankovic, Control of cascade systems with time delay—The integral cross-term approach, in: 2006 IEEE Conference on Decision and Control, San Diego, CA, 2006.
[28] I. Karafyllis, Finite-time global stabilization by means of time-varying distributed delay feedback, Siam Journal on Control and Optimization 45 (1) (2006) 320–342.
[29] I. Karafyllis, Z.P. Jiang, Control Lyapunov functionals and robust stabilization of nonlinear time-delay systems, in: 2008 IEEE Conference on Decision and Control, Cancun, Mexico, 2008.
[30] M. Krstic, A. Smyshlyaev, Backstepping boundary control for first-order hyperbolic PDEs and application to systems with actuator and sensor delays, Systems and Control Letters 57 (2008) 750–758.
[31] F. Mazenc, S. Mondie, S.I. Niculescu, Global asymptotic stabilization for chains of integrators with a delay in the input, IEEE Transactions on Automatic Control 48 (1) (2003) 57–63.
[32] F. Mazenc, S. Mondie, R. Francisco, Global asymptotic stabilization of feedforward systems with delay at the input, IEEE Transactions on Automatic Control 49 (2004) 844–850.
[33] F. Mazenc, P.-A. Bliman, Backstepping design for timedelay nonlinear systems, IEEE Transactions on Automatic Control 51 (2004) 149–154.
[34] L. Mirkin, On the approximation of distributed-delay control laws, Systems and Control Letters 51 (2004) 331–342.
[35] O.J.M. Smith, A controller to overcome dead time, ISA Transactions 6 (1959) 28–33.
[36] Q.-C. Zhong, L. Mirkin, Control of integral processes with dead time—Part 2: quantitative analysis, IEE Proceedings-Control Theory and Applications 149 (2002) 291–296.
[37] Q.-C. Zhong, On distributed delay in linear control laws—Part I: discrete-delay implementation, IEEE Transactions on Automatic Control 49 (2006) 2074–2080.
[38] Q.-C. Zhong, Robust Control of Time-delay Systems, Springer, 2006.