Compensation of Transport Actuator Dynamics With Input-Dependent Moving Controlled Boundary

Nikolaos Bekiaris-Liberis and Miroslav Krstic

***Abstract*—We introduce and solve the stabilization problem of a transport partial differential equation (PDE)/nonlinear ordinary differential equation (ODE) cascade, in which the PDE state evolves on a domain whose length depends on the boundary values of the PDE state itself. In particular, we develop a predictor-feedback control design, which compensates such transport PDE dynamics. We prove local asymptotic stability of the closed-loop system in the C^1 norm of the PDE state employing a Lyapunov-like argument and introducing a backstepping transformation. We also highlight the relation of the PDE–ODE cascade to a nonlinear system with input delay that depends on past input values and present the predictor-feedback control design for this representation as well.**

***Index Terms*—Predictor feedback, delay systems, distributed parameter systems, nonlinear systems.**

I. Introduction

Nonlinear systems with input delays that depend on the input itself can describe the dynamics of numerous physical processes. Among several other applications, such systems may model the dynamics of automotive engines [8], [19], [25], batch processes [9]–[11], [16], [17], blending processes [15], water heating processes [16], [38], production systems [21], chemical processes [24], [40], crushing mills [36], solar collectors [37], cooling systems [23] (where input-dependent delays appear due to the time required for the coolant to reach the consumers), and of vehicular traffic flow [26]. For this reason, it is of significant importance to develop control design methodologies for nonlinear systems with input-dependent input delays.

Prediction-based techniques have been successful in solving the stabilization problem of nonlinear systems with input delays that vary with time. In particular, prediction-based techniques are developed for the stabilization of systems with time-varying delays [3], [12], [29], [31]–[33], nonlinear systems with state-dependent delays [4]–[7], [20], [22], transport PDE/nonlinear ODE cascades with state-dependent moving boundaries [20], wave PDE/nonlinear ODE cascades with state-dependent moving boundaries [13], [14], and of nonlinear systems with input-dependent delays [8]–[11], [21]. However, the problem of stabilization of a transport PDE/nonlinear ODE cascade in which the PDE state evolves on a domain whose length depends on the boundary values of the PDE state itself has never been addressed.

Manuscript received September 11, 2017; revised January 11, 2018; accepted January 24, 2018. Date of publication February 1, 2018; date of current version October 25, 2018. The work of N. Bekiaris-Liberis was supported by the European Commission's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie Grant Agreement 747898, Project PADECOT. Recommended by Associate Editor W. Michiels. *(Corresponding author: Nikolaos Bekiaris-Liberis.)*

N. Bekiaris-Liberis is with the Department of Production Engineering and Management, Technical University of Crete, Chania 73100, Greece (e-mail: nikos.bekiaris@dssl.tuc.gr).

M. Krstic is with the Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093 USA (e-mail: krstic@ucsd.edu).

Digital Object Identifier 10.1109/TAC.2018.2800792

In this paper, we consider the stabilization problem of nonlinear systems with actuator dynamics governed by a transport PDE that evolves on a domain whose length depends on the boundary values of the PDE state itself. We develop a predictor-feedback control design methodology for the compensation of this type of actuator dynamics. The closed-loop system, under the predictor-feedback control law, is shown to be locally asymptotically stable, in the C^1 norm of the PDE state, via the employment of a Lyapunov-like argument and the introduction of a backstepping transformation. Our stability result is local due to an inherent limitation of the class of transport PDEs under consideration, which ensures the well-posedness of the given transport PDE. More specifically, this restriction guarantees that, in an equivalent formulation of the transport PDE that employs a constant PDE domain and a transport speed that depends on the boundary values of the PDE state as well as its first-order spatial derivative, the transport speed remains always strictly positive as well as uniformly bounded from above and below by finite constants.

Furthermore, we demonstrate that a special case of the considered transport PDE/nonlinear ODE cascade may be viewed as a nonlinear system with an input delay that is defined implicitly through a nonlinear equation, which involves the input value at a time that depends on the delay itself. This class of systems is different than the classes of systems considered in [11] and [21], in which, the input delays are defined implicitly via an integral equation that involves past input values. Note that the latter form of input delay is the result of the explicit dependence of the transport speed (rather than of the controlled boundary) on the boundary values of the PDE state.

Notation: We use the common definition of class $\mathcal{K}$, $\mathcal{K}_\infty$, and $\mathcal{KL}$ functions from [28]. For an n-vector, the norm $|\cdot|$ denotes the usual Euclidean norm. For scalar functions $u \in L^\infty[0, D(t)]$ or $v \in L^\infty[0,1]$ we denote by $\|u(t)\|_\infty$ or $\|v(t)\|_\infty$ their respective supremum norms, i.e., $\|u(t)\|_\infty = \sup_{x\in[0,D(t)]} |u(x,t)|$ or $\|v(t)\|_\infty = \sup_{z\in[0,1]} |v(z,t)|$. For scalar functions $u_x \in L^\infty[0, D(t)]$ or $v_z \in L^\infty[0,1]$ we denote by $\|u_x(t)\|_\infty$ or $\|v_z(t)\|_\infty$ their respective supremum norms, i.e., $\|u_x(t)\|_\infty = \sup_{x\in[0,D(t)]} |u_x(x,t)|$ or $\|v_z(t)\|_\infty = \sup_{z\in[0,1]} |v_z(z,t)|$. For vector valued functions $p \in L^\infty[0, D(t)]$ or $p_v \in L^\infty[0,1]$, we denote by $\|p(t)\|_\infty$ or $\|p_v(t)\|_\infty$ their respective supremum norms, i.e., $\|p(t)\|_\infty = \sup_{x\in[0,D(t)]} \sqrt{p_1(x,t)^2 + \cdots + p_n(x,t)^2}$ or $\|p_v(t)\|_\infty = \sup_{z\in[0,1]} \sqrt{p_1(z,t)^2 + \cdots + p_n(z,t)^2}$. For vector valued functions $p_x \in L^\infty[0, D(t)]$ or $p_{v\,z} \in L^\infty[0,1]$, we denote by $\|p_x(t)\|_\infty$ or $\|p_{v\,z}(t)\|_\infty$ their respective supremum norms, i.e., $\|p_x(t)\|_\infty = \sup_{x\in[0,D(t)]} \sqrt{p_{1\,x}(x,t)^2 + \cdots + p_{n\,x}(x,t)^2}$ or $\|p_{v\,z}(t)\|_\infty = \sup_{z\in[0,1]} \sqrt{p_{v\,1\,z}(z,t)^2 + \cdots + p_{v\,n\,z}(z,t)^2}$. We denote by $C^j(A;E)$ the space of functions that take values in E and have continuous derivatives of order j on A.

II. Problem Formulation and Predictor-Feedback Control Design

We consider the following system (see Fig. 1):

$$\dot{X}(t) = f\left(X(t), u(0,t)\right) \tag{1}$$

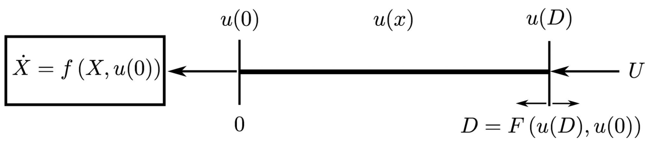

Fig. 1. Nonlinear system with actuator dynamics governed by a transport PDE, which evolves on a varying domain whose length depends on the boundary values of the PDE state itself.

$$u_t(x,t) = u_x(x,t) \tag{2}$$

$$u(D(t), t) = U(t) \tag{3}$$

where $X \in \mathbb{R}^n$ is the ODE state, $t \geq 0$ is time, $x \in [0, D(t)]$ is spatial variable, $U \in C^1[0, \infty)$ is a scalar control input, u is the PDE state of the actuator dynamics, $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a continuously differentiable vector field that satisfies $f(0,0) = 0$, and D is a moving boundary that is defined as

$$D(t) = F(u(D(t), t), u(0,t)). \tag{4}$$

The following assumptions are imposed on system (1)–(4).

Assumption 1: Function $F : \mathbb{R}^2 \to \mathbb{R}$ is twice continuously differentiable and satisfies

$$F(u_1, u_2) > 0, \quad \text{for all } (u_1, u_2) \in \mathbb{R}^2. \tag{5}$$

Assumption 2: System $\dot{X} = f(X, \omega)$ is strongly forward complete with respect to ω.

Assumption 3: There exists a twice continuously differentiable feedback law $\kappa : \mathbb{R}^n \to \mathbb{R}$, with $\kappa(0) = 0$, which renders system $\dot{X} = f(X, \kappa(X) + \omega)$ input-to-state stable with respect to ω.

Assumption 1 is a mild assumption on the moving boundary function F, which ensures that the transport equation (2), (3) is meaningful. Assumption 2 (see, e.g., [1]) guarantees that for every initial condition and every locally bounded input signal, the corresponding solution of (1) is defined for all $t \geq 0$. Hence, it implies that the state X of system (1) does not escape to infinity before the control signal U reaches it, no matter the size of the delay (see, e.g., [5], [29], [30]). Assumption 3 (see, e.g., [39]) guarantees the existence of a nominal feedback law that renders system (1) input-to-state stable in the absence of the transport actuator dynamics (i.e., in the absence of the input delay). This assumption is a standard ingredient of the predictor-feedback control design methodology (see, e.g., [5], [29], [30]).

The predictor-feedback control law for system (1)–(3) is given by

$$U(t) = \kappa(p(D(t), t)) \tag{6}$$

where for all $x \in [0, D(t)]$ and $t \geq 0$

$$p(x,t) = X(t) + \int_0^x f(p(y,t), u(y,t))\, dy. \tag{7}$$

For the implementation of the predictor-feedback law (6), (7) it is required that the ODE state $X(t)$ and the PDE state $u(x,t)$, $x \in [0, D(t)]$ are measured for all $t \geq 0$. Note that the position of the moving boundary $D(t)$, for all $t \geq 0$, can be computed at each time instant t employing the right-hand side of expression (4) and the boundary measurements of the PDE state, unless it is directly measured. It is worth mentioning here that the implementation problem of predictor-feedback control laws is tackled in several works, such as, for example, [27], [34], [41].

For the subsequent analysis it turns out that it is useful to transform the PDE (2), (3), which evolves on a varying domain, to a PDE that evolves on a constant domain. Defining

$$x = D(t) z \tag{8}$$

$$v(z,t) = u(D(t) z, t) \tag{9}$$

we rewrite (1)–(3) as

$$\dot{X}(t) = f(X(t), v(0,t)) \tag{10}$$

$$v_t(z,t) = \frac{1 + z \dfrac{\nabla F(v(1,t), v(0,t)) \frac{(v_z(1,t), v_z(0,t))^{\mathrm{T}}}{F(v(1,t), v(0,t))}}{1 - F_{u_1}(v(1,t), v(0,t)) \frac{v_z(1,t)}{F(v(1,t), v(0,t))}}}{F(v(1,t), v(0,t))} v_z(z,t)$$

$$z \in [0,1] \tag{11}$$

$$v(1,t) = U(t). \tag{12}$$

In order to guarantee the well-posedness of the transport PDE (11), (12) the transport speed must be strictly positive as well as uniformly bounded from above and below. Since the transport speed depends on the PDE state itself, the following conditions on the closed-loop solutions and the initial conditions it is needed to be satisfied for all $t \geq 0$

$$1 - \epsilon_1 < \frac{v_z(1,t) F_{u_1}(v(1,t), v(0,t))}{F(v(1,t), v(0,t))} < 1 - \epsilon_2 \tag{13}$$

$$\epsilon_3 - 1 < \frac{v_z(0,t) F_{u_2}(v(1,t), v(0,t))}{F(v(1,t), v(0,t))} < \epsilon_4 - 1 \tag{14}$$

$$\epsilon_5 < F(v(1,t), v(0,t)) < \epsilon_6 \tag{15}$$

for some positive constants ϵ_1, ϵ_2, ϵ_3, ϵ_4, ϵ_5, and ϵ_6, which may depend on the function F (see also relations (A64)–(A68) and (A75)–(A78) in the Appendix for more details).

III. Stability Analysis

Theorem 1: Consider the closed-loop system consisting of the plant (1)–(4) and the control law (6), (7). Under Assumptions 1, 2, and 3, there exist a positive constant δ_u and a class $\mathcal{KL}$ function β_u such that for all initial conditions $X(0) \in \mathbb{R}^n$ and $u(\cdot, 0) \in C^1[0, D(0)]$ which satisfy

$$|X(0)| + \|u(0)\|_\infty + \|u_x(0)\|_\infty < \delta_u \tag{16}$$

as well as the compatibility conditions

$$u(D(0), 0) = \kappa(p(D(0), 0)) \tag{17}$$

$$u_x(D(0), 0) = \frac{\partial \kappa(p(D(0), 0))}{\partial p} \times f(p(D(0), 0), u(D(0), 0)) \tag{18}$$

the following holds:

$$\Omega(t) \leq \beta_u(\Omega(0), t), \quad \text{for all } t \geq 0 \tag{19}$$

$$\Omega(t) = |X(t)| + \|u(t)\|_\infty + \|u_x(t)\|_\infty. \tag{20}$$

The proof of Theorem 1 is based on the following lemmas whose proofs can be found in the Appendix. The first two lemmas introduce a backstepping transformation, together with its inverse, which transforms the original closed-loop system (10)–(12), (6), (7) into a suitable "target system" whose stability properties can be established.

Lemma 1: Consider the backstepping transformation

$$w(z,t) = v(z,t) - \kappa(p_v(z,t)) \tag{21}$$

where

$$p_v(z,t) = X(t) + F(v(1,t), v(0,t)) \times \int_0^z f(p_v(s,t), v(s,t))\, ds \tag{22}$$

together with its inverse

$$v(z,t) = w(z,t) + \kappa(\pi_v(z,t)) \tag{23}$$

where

$$\pi_v(z,t) = X(t) + F(\kappa(\pi_v(1,t)), w(0,t) + \kappa(X(t))) \int_0^z f(\pi_v(y,t), w(y,t) + \kappa(\pi_v(y,t)))\, dy. \tag{24}$$

Transformation (21) together with the control law (6), (7) transform system (10)–(12) to the following target system:

$$\dot{X}(t) = f(X(t), \kappa(X(t)) + w(0,t)) \tag{25}$$

$$w_t(z,t) = \xi(z,t) w_z(z,t) \tag{26}$$

$$w(1,t) = 0 \tag{27}$$

where

$$\xi(z,t) = \frac{1 + z \frac{\nabla F(v(1,t), v(0,t)) \frac{(v_z(1,t), v_z(0,t))^{\mathrm{T}}}{F(v(1,t), v(0,t))}}{1 - F_{u_1}(v(1,t), v(0,t)) \frac{v_z(1,t)}{F(v(1,t), v(0,t))}}}{F(v(1,t), v(0,t))} \tag{28}$$

with $v(0,t)$ expressed in terms of $w(0,t)$ and $X(t)$ using (23), (24) for $z = 0$ and $v(1,t)$ expressed in terms of $\pi_v(1,t)$ using (23), (24) for $z = 1$ and (27).

The next lemma shows that the target system (25)–(27) is asymptotically stable.

Lemma 2: There exists a class $\mathcal{KL}$ function β_w such that for all solutions of the system satisfying (13)–(15) the following holds:

$$\Omega_w(t) \leq \beta_w(\Omega_w(0), t) \quad \text{for all } t \geq 0 \tag{29}$$

$$\Omega_w(t) = |X(t)| + \|w(t)\|_\infty + \|w_z(t)\|_\infty. \tag{30}$$

Lemmas 3–5 establish the norm equivalency between the original system (10)–(12) and the target system (25)–(27).

Lemma 3: There exists a class $\mathcal{K}_\infty$ function ρ_1 such that the following holds for all $t \geq 0$:

$$\|p_v(t)\|_\infty + \|p_{v\,z}(t)\|_\infty \leq \rho_1(|X(t)| + \|v(t)\|_\infty). \tag{31}$$

Lemma 4: There exists a class $\mathcal{K}_\infty$ function ρ_2 such that the following holds for all $t \geq 0$:

$$\|\pi_v(t)\|_\infty + \|\pi_{v\,z}(t)\|_\infty \leq \rho_2(|X(t)| + \|w(t)\|_\infty). \tag{32}$$

Lemma 5: There exist class $\mathcal{K}_\infty$ functions ρ_3 and ρ_4 such that the following hold for all $t \geq 0$:

$$\Omega_w(t) \leq \rho_3(\Omega_v(t)) \tag{33}$$

$$\Omega_v(t) \leq \rho_4(\Omega_w(t)) \tag{34}$$

where Ω_w is defined in (30) and

$$\Omega_v(t) = |X(t)| + \|v(t)\|_\infty + \|v_z(t)\|_\infty. \tag{35}$$

The next two lemmas show the equivalency of the C^1 norm of state $(X, u(x))$, $x \in [0, D(t)]$ to the state $(X, v(z))$, $z \in [0,1]$.

Lemma 6: There exists a class $\mathcal{K}_\infty$ function ρ_5 such that the following holds for all $t \geq 0$:

$$\Omega_v(t) \leq \rho_5(\Omega(t)) \tag{36}$$

where Ω_v and Ω are defined in (35) and (20), respectively.

Lemma 7: There exists a class $\mathcal{K}_\infty$ function ρ_6 such that for all solutions of the system satisfying (15) the following holds for all $t \geq 0$:

$$\Omega(t) \leq \rho_6(\Omega_v(t)) \tag{37}$$

where Ω and Ω_v are defined in (20) and (35), respectively.

In the last three lemmas, an estimate of the region of attraction of the predictor-feedback control law (6), (7) is provided.

Lemma 8: There exists a positive constant δ_1 such that for all solutions of the system that satisfy

$$|X(t)| + \|v(t)\|_\infty + \|v_z(t)\|_\infty < \delta_1 \quad \text{for all } t \geq 0 \tag{38}$$

they also satisfy (13)–(15).

Lemma 9: There exists a positive constant δ_u such that for all initial conditions of the closed-loop system (1)–(3), (6), (7) that satisfy (16) the solutions of the system satisfy (38), and hence, satisfy (13)–(15).

Proof of Theorem 1: Theorem 1 is proved combining Lemmas 2, 5, 6, and 7 with

$$\beta_u(s,t) = \rho_6(\rho_4(\beta_w(\rho_3(\rho_5(s)), t))). \tag{39}$$

IV. Relation to a System With Delayed-Input-Dependent Input Delay

Consider the following system:

$$\dot{X}(t) = f(X(t), U(\phi(t))) \tag{40}$$

where the delayed time ϕ is defined implicitly through relation

$$\phi(t) = t - F(U(\phi(t))) \tag{41}$$

and $F : \mathbb{R} \to \mathbb{R}_+$ is a delay. In fact, system (40) and (41) is an equivalent delay system representation of system (1)–(4), where for simplicity of presentation we consider only the dependence of F from $U(\phi)$. To see this, note that the solution to (2) and (3) is given for all $x \in [0, D(t)]$ and $t \geq 0$ by

$$u(x,t) = U(\phi(t+x)) \tag{42}$$

where the prediction time σ, i.e., the inverse function of ϕ, is given by

$$\sigma(t) = t + D(t) = t + F(U(t)). \tag{43}$$

The predictor-feedback control law for system (40) with an input delay defined via (41) is given by

$$U(t) = \kappa(P(t)) \tag{44}$$

where the predictor P is given for all $t \geq 0$ by

$$P(\theta) = X(t) + \int_{\phi(t)}^{\theta} \left(1 + F'(U(s))\dot{U}(s)\right) \times f(P(s), U(s))\, ds, \quad \text{for all } \phi(t) \leq \theta \leq t. \tag{45}$$

The predictor-feedback control law is implementable since it depends on the history of $U(s)$ and $\dot{U}(s)$, over the window $\phi(t) \leq s \leq t$, as well as on the ODE state $X(t)$, which are assumed to be measured for all $t \geq 0$. Moreover, the implementation of the predictor-feedback design requires the computation at each time step of the delayed time ϕ. This can either be performed by numerically solving relation (41) or by employing the following integral equation:

$$\phi(\theta) = t - \int_{\theta}^{\sigma(t)} \frac{ds}{1 + F'(U(\phi(s)))U'(\phi(s))} \quad \text{for all } t \leq \theta \leq \sigma(t) \tag{46}$$

where σ is defined in (43). Note that the key condition for the well-posedness of system (40), (41), and the predictor-feedback control design (44), (45) is reflected by the need to keep the denominator in (46) positive.

V. Conclusion

We introduced a predictor-feedback control design methodology for nonlinear systems with transport actuator dynamics, which evolve on a varying domain whose length depends on the boundary values of the transport PDE sate. We proved local asymptotic stability of the closed-loop system under predictor-feedback in the C^1 norm of the actuator state, employing a Lyapunov-like argument and a novel backstepping transformation. The relation of the PDE-ODE cascade to a nonlinear system with input delay that depends on past input values was also highlighted and the predictor-feedback control design for this representation was also presented.

Appendix

Proof of Lemma 1

Performing in the integral in (7), the change of variables $y = D(t)s$ and using the fact that F is positive (Assumption 1), we obtain

$$p(x,t) = X(t) + D(t) \times \int_0^{\frac{x}{D(t)}} f\left(p\left(D(t)s,t\right), u\left(D(t)s,t\right)\right) ds. \quad \text{(A1)}$$

Therefore, with definitions (8) and (9), and

$$p_v(z,t) = p\left(D(t)z,t\right) \quad \text{for all } z \in [0,1], \quad \text{(A2)}$$

the control law (6) and (7) is expressed in terms of the v variable as

$$U(t) = \kappa\left(p_v\left(1,t\right)\right) \quad \text{(A3)}$$

where p_v is given by (22). The function p satisfies

$$p_t(x,t) = p_x(x,t), \quad x \in [0,1] \quad \text{(A4)}$$

which can be shown by noting that u satisfies (2). Therefore, using (2), (4) and definitions (9), (2), we obtain

$$w_t(z,t) = \left(z\dot{D}(t)+1\right)\left(-\frac{\partial\kappa\left(p_v(z,t)\right)}{\partial p_v} p_x\left(D(t)z,t\right) + u_x\left(D(t)z,t\right)\right) \quad \text{(A5)}$$

$$w_z(z,t) = D(t)\left(u_x\left(D(t)z,t\right) - \frac{\partial\kappa\left(p_v(z,t)\right)}{\partial p_v} p_x\left(D(t)z,t\right)\right). \quad \text{(A6)}$$

Hence,

$$w_t(z,t) = \frac{z\dot{D}(t)+1}{D(t)} w_z(z,t). \quad \text{(A7)}$$

Moreover, using (2) and (4), it follows that

$$\frac{1+z\dot{D}(t)}{D(t)} = \frac{1 + z\frac{\nabla F\left(v(1,t),v(0,t)\right)\frac{\left(v_z(1,t),v_z(0,t)\right)^{\mathrm{T}}}{F\left(v(1,t),v(0,t)\right)}}{1-F_{u_1}\left(v(1,t),v(0,t)\right)\frac{v_z(1,t)}{F\left(v(1,t),v(0,t)\right)}}}{F\left(v(1,t),v(0,t)\right)}. \quad \text{(A8)}$$

Substituting on the right-hand side of (A8), the terms $v(1,t)$, $v(0,t)$, $v_z(0,t)$, and $v_z(1,t)$ via relation (23), we arrive at (26). Finally, relation (27) follows by setting $z = 1$ into (21) and using (3) and (6) as well as definitions (9) and (A2) for $z = 1$.

Proof of Lemma 2

By (13)–(15), it holds that

$$\frac{\min\left\{1, \frac{\epsilon_3}{\epsilon_1}\right\}}{\epsilon_6} \le \xi(z,t) \le \frac{\min\left\{1, \frac{\epsilon_4}{\epsilon_2}\right\}}{\epsilon_5} \quad \text{(A9)}$$

for all $z \in [0,1]$ and $t \ge 0$, where ξ is defined in (28). Moreover, from (26), (27), and (A9), it follows that

$$w_{zt}(z,t) = \xi_z(z,t)w_z(z,t) + \xi(z,t)w_{zz} \quad \text{(A10)}$$

$$w_z(1,t) = 0. \quad \text{(A11)}$$

Consider now the following Lyapunov functional:

$$L_{c,m}(t) = \int_0^1 e^{2(c+\lambda)zm} w(z,t)^{2m} dz + \int_0^1 e^{2(c+\lambda)zm} w_z(z,t)^{2m} dz \quad \text{(A12)}$$

for any $c > 0$ and any positive integer m. Taking the time derivative of (A12) along the solutions of (25)–(27), (A10), and (A11), we get using integration by parts definition (28) and (A9)

$$\dot{L}_{c,m}(t) \le -\int_0^1 e^{2(c+\lambda)zm} w(z,t)^{2m} \left(2m(c+\lambda)\xi(z,t) + \xi_z(z,t)\right) dz - \int_0^1 e^{2(c+\lambda)zm} w_z(z,t)^{2m} \left(2m \times (c+\lambda)\xi(z,t) + \xi_z(z,t) - 2m\xi_z(z,t)\right) dz. \quad \text{(A13)}$$

From the definition of ξ in (28), one can observe that ξ is a linear function of z, and hence, the functions $2m(c+\lambda)\xi(z,t) + \xi_z(z,t) - 2m\xi_z(z,t)$ and $2m(c+\lambda)\xi(z,t) + \xi_z(z,t)$ attain their minimum value either at $z = 0$ or $z = 1$. Using this fact together with (13)–(15), we arrive at

$$2m(c+\lambda)\xi(z,t) + \xi_z(z,t) \ge 2mc\epsilon \quad \text{(A14)}$$

$$2m(c+\lambda)\xi(z,t) + \xi_z(z,t)(1-2m) \ge 2mc\epsilon \quad \text{(A15)}$$

where

$$\epsilon = \frac{\min\{\epsilon_2, \epsilon_3\}}{\epsilon_1\epsilon_6} \quad \text{(A16)}$$

whenever

$$\lambda \ge \frac{|\epsilon_1 - \epsilon_3|}{\min\{\epsilon_2, \epsilon_3\}}. \quad \text{(A17)}$$

Thus,[1]

$$\dot{L}_{c,m}(t) \le -2mc\epsilon L_{c,m}(t) \quad \text{(A18)}$$

which implies that

$$L_{c,m}^{\frac{1}{2m}}(t) \le e^{-c\epsilon(t-s)} L_{c,m}^{\frac{1}{2m}}(s) \quad \text{for all } t \ge s. \quad \text{(A19)}$$

Moreover, from (A12), it follows that:

$$\Xi_{c,m}(t) \le 2e^{-c\epsilon(t-s)}\Xi_{c,m}(s) \quad \text{(A20)}$$

[1]Note that although the estimate (A18) for $L_{c,m}$ is derived for v that is of class C^2 [and thus, so is w satisfying (A10) and (A11)], adopting the arguments from, e.g., [13], the estimate (A18) remains valid (in the distribution sense) when v is only of class C^1.

where

$$\Xi_{c,m}(t) = \left(\int_0^1 e^{2(c+\lambda)zm} w(z,t)^{2m}\, dz\right)^{\frac{1}{2m}} + \left(\int_0^1 e^{2(c+\lambda)zm} w_z(z,t)^{2m}\, dz\right)^{\frac{1}{2m}}. \quad \text{(A21)}$$

Taking the limit of (A21) as m goes to infinity, with the definition of the supremum norm, i.e., with relation $\|\theta(t)\|_\infty = \lim_{m\to\infty} (\int_0^1 |\theta(z,t)|^{2m}\, dz)^{\frac{1}{2m}}$, we obtain

$$\Xi_c(t) \le 2e^{-c\epsilon(t-s)}\Xi_c(s) \quad \text{(A22)}$$

where

$$\Xi_c(t) = \sup_{0\le z\le 1}\left|e^{z(c+\lambda)}w(z,t)\right| + \sup_{0\le z\le 1}\left|e^{z(c+\lambda)}w_z(z,t)\right|. \quad \text{(A23)}$$

It follows, for all $t \ge s$, that

$$\|w(t)\|_\infty + \|w_z(t)\|_\infty \le 2e^{-c\epsilon(t-s)}e^{(c+\lambda)}\left(\|w(s)\|_\infty + \|w_z(s)\|_\infty\right). \quad \text{(A24)}$$

Under Assumption 3 (see, e.g., [39]), we obtain from (25) that

$$|X(t)| \le \beta_1\left(|X(s)|, t-s\right) + \gamma_1\left(\sup_{s\le\tau\le t}\|w(\tau)\|\right) \quad \text{(A25)}$$

for all $t \ge s \ge 0$, some class $\mathcal{KL}$ function β_1, and some class $\mathcal{K}$ function γ_1. Mimicking the arguments in [28, Proof of Lemma 4.7], we set $s = \frac{t}{2}$ in (A25) to obtain

$$|X(t)| \le \beta_1\left(\left|X\left(\frac{t}{2}\right)\right|, \frac{t}{2}\right) + \gamma_1\left(\sup_{\frac{t}{2}\le\tau\le t}\|w(\tau)\|\right) \quad \text{(A26)}$$

and thus, using (A25) for $s = 0$ and $t \to \frac{t}{2}$, we arrive at

$$|X(t)| \le \beta_1\left(\beta_1\left(|X(0)|, \frac{t}{2}\right) + \gamma_1\left(\sup_{0\le\tau\le\frac{t}{2}}\|w(\tau)\|\right), \frac{t}{2}\right) + \gamma_1\left(\sup_{\frac{t}{2}\le\tau\le t}\|w(\tau)\|\right) \quad \text{for all } t \ge 0. \quad \text{(A27)}$$

Moreover, using (A24), we obtain

$$\sup_{0\le\tau\le\frac{t}{2}}\|w(\tau)\| \le 2e^{(c+\lambda)}\left(\|w(0)\|_\infty + \|w_z(0)\|_\infty\right) \quad \text{(A28)}$$

$$\sup_{\frac{t}{2}\le\tau\le t}\|w(\tau)\| \le 2e^{-c\epsilon\frac{t}{2}}e^{(c+\lambda)} \times\left(\|w(0)\|_\infty + \|w_z(0)\|_\infty\right). \quad \text{(A29)}$$

Therefore, combining (A27) with (A28) and (A29) and using (A24), we get (29) with

$$\beta_w(s,t) = \beta_1\left(\beta_1(s,0) + \gamma_1\left(2e^{(c+\lambda)}s\right), \frac{t}{2}\right) + 2e^{-c\epsilon t} \times e^{(c+\lambda)}s + \gamma_1\left(2e^{-c\epsilon\frac{t}{2}}e^{(c+\lambda)}s\right). \quad \text{(A30)}$$

Proof of Lemma 3

From (22), it follows that

$$p_{v\,z}(z,t) = F\left(v(1,t), v(0,t)\right) f\left(p_v(z,t), v(z,t)\right). \quad \text{(A31)}$$

Under Assumption 2, there exists a smooth function $R : \mathbb{R}^n \to \mathbb{R}_+$ and class $\mathcal{K}_\infty$ functions α_1, α_2, and α_3 such that (see, e.g., [1], [29], and [30])

$$\alpha_1(|X|) \le R(X) \le \alpha_2(|X|) \quad \text{(A32)}$$

$$\frac{\partial R(X)}{\partial X} f(X,\omega) \le R(X) + \alpha_3(|\omega|) \quad \text{(A33)}$$

for all $(X,\omega)^{\mathrm{T}} \in \mathbb{R}^{n+1}$. Thus, from (A31) and (A33), we get under the positivity assumption of F (Assumption 1) that

$$\frac{\partial R(p_v(z,t))}{\partial p_v} p_{v\,z}(z,t) \le F\left(v(1,t), v(0,t)\right)\left(R\left(p_v(z,t)\right) + \alpha_3\left(|v(z,t)|\right)\right). \quad \text{(A34)}$$

Therefore, employing the comparison principle and using the fact that $p_v(0,t) = X(t)$, we obtain

$$R\left(p_v(z,t)\right) \le e^{F(v(1,t),v(0,t))}R\left(X(t)\right) + F\left(v(1,t), v(0,t)\right) \times \int_0^z e^{F(v(1,t),v(0,t))(z-y)}\alpha_3(|v(y,t)|)dy. \quad \text{(A35)}$$

Under Assumption 1 (continuity and positiveness of F), we conclude that there exists a class $\mathcal{K}_\infty$ function $\hat{\alpha}$ such that

$$F\left(v(1,t), v(0,t)\right) \le F(0,0) + \hat{\alpha}\left(|v(1,t)| + |v(0,t)|\right) \quad \text{(A36)}$$

and, hence,

$$F\left(v(1,t), v(0,t)\right) \le F(0,0) + \hat{\alpha}\left(2\|v(t)\|_\infty\right). \quad \text{(A37)}$$

Consequently, employing (A32) and (A37), we get from (A35) that

$$\|p_v(t)\|_\infty \le \alpha_4\left(|X(t)| + \|v(t)\|_\infty\right) \quad \text{(A38)}$$

where

$$\alpha_4(s) = \alpha_1^{-1}\left(e^{F(0,0)+\hat{\alpha}(s)}\left(\alpha_2(s) + \alpha_3(s)\right)\right). \quad \text{(A39)}$$

Since f is continuously differentiable with $f(0,0) = 0$, we conclude that there exists a class $\mathcal{K}_\infty$ function α_5 such that

$$|f(X,\omega)| \le \alpha_5\left(|X| + |\omega|\right). \quad \text{(A40)}$$

Thus, using (A31), (A37), and (A38), we arrive at

$$|p_{v\,z}(z,t)| \le \alpha_6\left(|X(t)| + \|v(t)\|_\infty\right) \quad \forall z \in [0,1] \quad \text{(A41)}$$

where

$$\alpha_6(s) = \left(F(0,0) + \hat{\alpha}(2s)\right)\alpha_5\left(\alpha_4(s) + s\right). \quad \text{(A42)}$$

The proof is completed by taking a supremum in both sides of (A41) and setting $\rho_1(s) = \alpha_4(s) + \alpha_6(s)$.

Proof of Lemma 4

From (24), it follows that

$$\pi_{vz}(z,t) = F\left(\kappa\left(\pi_v(1,t)\right), w(0,t) + \kappa\left(X(t)\right)\right) \times f\left(\pi_v(z,t), w(z,t) + \kappa\left(\pi_v(z,t)\right)\right) \quad \text{(A43)}$$

and thus, defining

$$\pi(x,t) = \pi\left(D(t)z,t\right) \equiv \pi_v\left(z,t\right) \quad \text{(A44)}$$

and

$$w_u(x,t) = w_u\left(D(t)z,t\right) \equiv w\left(z,t\right) \quad \text{(A45)}$$

we get using (27) and (23) that for all $x \in [0, D(t)]$

$$\pi_x(x,t) = f\left(\pi(x,t), w_u(x,t) + \kappa\left(\pi(x,t)\right)\right). \quad \text{(A46)}$$

Under Assumption 3 (see, e.g., [39]) and using the fact that $\pi(0,t) = X(t)$, which follows from (24), (A44) for $z = 0$, there exists a class $\mathcal{KL}$ function $\hat{\beta}$ and a class $\mathcal{K}_\infty$ function ζ such that

$$|\pi(x,t)| = \hat{\beta}\left(|X(t)|, x\right) + \zeta\left(\sup_{0\le r\le x}|w_u(r,t)|\right). \quad \text{(A47)}$$

Thus,

$$\|\pi(t)\|_\infty \le \hat{\beta}\left(|X(t)|, 0\right) + \zeta\left(\sup_{0\le r\le D(t)}|w_u(r,t)|\right) \quad \text{(A48)}$$

and hence, with definitions (A44) and (A45), we obtain

$$\|\pi_v(t)\|_\infty \le \hat{\beta}\left(|X(t)|, 0\right) + \zeta\left(\|w(t)\|_\infty\right). \quad \text{(A49)}$$

Under Assumption 3 (continuity of κ and the fact that $\kappa(0) = 0$), there exists a class $\mathcal{K}_\infty$ function $\hat{\hat{\alpha}}_1$ such that

$$|\kappa\left(X\right)| \le \hat{\hat{\alpha}}_1\left(|X|\right). \quad \text{(A50)}$$

Under Assumption 1 and (A36), we conclude that there exists a class $\mathcal{K}_\infty$ function $\hat{\alpha}$ such that

$$F\left(\kappa\left(\pi_v(1,t)\right), w(0,t) + \kappa\left(X(t)\right)\right) \le F(0,0) + \hat{\alpha}\left(|\kappa\left(\pi_v(1,t)\right)| + |w(0,t)| + |\kappa\left(X(t)\right)|\right) \quad \text{(A51)}$$

and thus, using (A50) and (A49), we obtain

$$F\left(\kappa\left(\pi_v(1,t)\right), w(0,t) + \kappa\left(X(t)\right)\right) \le F(0,0) + \hat{\alpha}\left(\hat{\hat{\alpha}}_1\left(\hat{\beta}\left(|X(t)|, 0\right) + \zeta\left(\|w(t)\|_\infty\right)\right) + \|w(t)\|_\infty + \hat{\hat{\alpha}}_1\left(|X(t)|\right)\right). \quad \text{(A52)}$$

Therefore, from (A43) using (A40) and (A52), we obtain

$$|\pi_{vz}(z,t)| \le \alpha_7\left(|X(t)| + \|w(t)\|_\infty\right) \quad \text{(A53)}$$

where

$$\alpha_7(s) = \left(F(0,0) + \hat{\alpha}\left(\hat{\hat{\alpha}}_1\left(\hat{\beta}\left(s,0\right) + \zeta\left(s\right)\right) + s + \hat{\hat{\alpha}}_1\left(s\right)\right)\right) \times \alpha_5\left(\hat{\beta}\left(s,0\right) + \zeta(s) + s + \hat{\hat{\alpha}}_1\left(\hat{\beta}\left(s,0\right) + \zeta(s)\right)\right). \quad \text{(A54)}$$

Proof of Lemma 5

Under Assumption 3 (continuous differentiability of κ), there exists a class $\mathcal{K}_\infty$ function $\hat{\hat{\alpha}}_2$ such that

$$|\nabla\kappa\left(X\right)| \le |\nabla\kappa\left(0\right)| + \hat{\hat{\alpha}}_2\left(|X|\right) \quad \text{(A55)}$$

for all $X \in \mathbb{R}^n$. Therefore, using (31) and (A50), we get from (21) that

$$|w(z,t)| + |w_z(z,t)| \le |v(z,t)| + |v_z(z,t)| + \hat{\hat{\alpha}}_1\left(\rho_1\left(|X(t)| + \|v(t)\|_\infty\right)\right) + \left(|\nabla\kappa\left(0\right)| + \hat{\hat{\alpha}}_2\left(\rho_1\left(|X(t)| + \|v(t)\|_\infty\right)\right)\right) \times \rho_1\left(|X(t)| + \|v(t)\|_\infty\right) \quad \text{(A56)}$$

and thus, estimate (33) follows with

$$\rho_3\left(s\right) = s + \hat{\hat{\alpha}}_1\left(\rho_1\left(s\right)\right) + \left(|\nabla\kappa\left(0\right)| + \hat{\hat{\alpha}}_2\left(\rho_1\left(s\right)\right)\right)\rho_1\left(s\right). \quad \text{(A57)}$$

Similarly, using (A50) and (A55) and combining (23) with (32), estimate (34) follows with

$$\rho_4\left(s\right) = s + \hat{\hat{\alpha}}_1\left(\rho_2\left(s\right)\right) + \left(|\nabla\kappa\left(0\right)| + \hat{\hat{\alpha}}_2\left(\rho_2\left(s\right)\right)\right)\rho_2\left(s\right). \quad \text{(A58)}$$

Proof of Lemma 6

Using (A36), we get that

$$\sup_{z\in[0,1]}|v_z(z,t)| = \sup_{x\in[0,D(t)]}|u_x(x,t)F\left(u(D(t),t), u(0,t)\right)| \le \|u_x(t)\|_\infty\left(F(0,0) + \hat{\alpha}\left(2\|u(t)\|_\infty\right)\right). \quad \text{(A59)}$$

Thus,

$$\Omega_v(t) \le |X(t)| + \|u(t)\|_\infty + \|u_x(t)\|_\infty \times \left(F(0,0) + \hat{\alpha}\left(2\|u(t)\|_\infty\right)\right) \quad \text{(A60)}$$

where Ω_v is defined in (35). Using (A60), we get (36) with

$$\rho_5\left(s\right) = s + s\left(F(0,0) + \hat{\alpha}\left(2s\right)\right). \quad \text{(A61)}$$

Proof of Lemma 7

Utilizing the fact that

$$|u_x\left(D(t)z,t\right)| = \frac{|v_z(z,t)|}{F\left(v(1,t), v(0,t)\right)} \quad \text{(A62)}$$

from the left-hand side of inequality (15), we obtain

$$\|u_x(t)\|_\infty \le \frac{1}{\epsilon_5}\|v_z(t)\|_\infty. \quad \text{(A63)}$$

Therefore, relation (37) is obtained with $\rho_6(s) = (1 + \frac{1}{\epsilon_5})s$.

Proof of Lemma 8

Under Assumption 1 and (A36), one can conclude that the following holds:

$$F(0,0) - \hat{\alpha}\left(|v(1,t)| + |v(0,t)|\right) \le F\left(v(1,t), v(0,t)\right). \quad \text{(A64)}$$

Taking any positive constant δ_1 such that

$$\delta_1 < \hat{\alpha}_1^{-1}\left(F(0,0)\right) \quad \text{(A65)}$$

where $\hat{\alpha}_1(s) = \hat{\alpha}(2s)$. It follows from (A36) and (A64) that condition (15) is satisfied with any choice of ϵ_5, ϵ_6 such that

$$0 < \epsilon_5 \leq F(0,0) - \hat{\alpha}(2\delta_1) \quad \text{(A66)}$$

$$\epsilon_6 \geq F(0,0) + \hat{\alpha}(2\delta_1). \quad \text{(A67)}$$

Moreover, under Assumption 1 (continuous differentiability of F), we conclude that there exists a class $\mathcal{K}_\infty$ function $\hat{\rho}$ such that

$$|\nabla F(v(1,t), v(0,t))| \leq |\nabla F(0,0)| + \hat{\rho}(|v(1,t)| + |v(0,t)|). \quad \text{(A68)}$$

Using (A64), (65), (68), and (38), we have

$$\Gamma(\delta_1) > |\Gamma_1(v(1,t), v(0,t), v_z(0,t))| + |\Gamma_2(v(1,t), v(0,t), v_z(1,t))| \quad \text{(A69)}$$

$$\Gamma(\delta_1) = \frac{2\delta_1(|\nabla F(0,0)| + \hat{\rho}(2\delta_1))}{F(0,0) - \hat{\alpha}_1(\delta_1)} \quad \text{(A70)}$$

$$\Gamma_1(v(1,t), v(0,t), v_z(0,t)) = \frac{v_z(0,t) F_{u_2}(v(1,t), v(0,t))}{F(v(1,t), v(0,t))} \quad \text{(A71)}$$

$$\Gamma_2(v(1,t), v(0,t), v_z(1,t)) = \frac{v_z(1,t) F_{u_1}(v(1,t), v(0,t))}{F(v(1,t), v(0,t))}. \quad \text{(A72)}$$

Thus,

$$-\Gamma(\delta_1) < \Gamma_2(v(1,t), v(0,t), v_z(1,t)) < \Gamma(\delta_1) \quad \text{(A73)}$$

and

$$-\Gamma(\delta_1) < \Gamma_1(v(1,t), v(0,t), v_z(0,t)) < \Gamma(\delta_1). \quad \text{(A74)}$$

Hence, conditions (13) and (14) are satisfied with any choice of positive constants $\epsilon_1, \epsilon_2, \epsilon_3$, and ϵ_4 such that

$$\epsilon_1 = \epsilon_4 \geq 1 + \frac{2\delta_1(|\nabla F(0,0)| + \hat{\rho}(2\delta_1))}{F(0,0) - \hat{\alpha}_1(\delta_1)} \quad \text{(A75)}$$

$$\epsilon_2 = \epsilon_3 \leq 1 - \frac{2\delta_1(|\nabla F(0,0)| + \hat{\rho}(2\delta_1))}{F(0,0) - \hat{\alpha}_1(\delta_1)} \quad \text{(A76)}$$

whenever

$$\delta_1 < \bar{\alpha}^{-1}(F(0,0)) \quad \text{(A77)}$$

where the class $\mathcal{K}_\infty$ function $\bar{\alpha}$ is given by

$$\bar{\alpha}(s) = \hat{\alpha}(2s) + 2s(|\nabla F(0,0)| + \hat{\rho}(2s)). \quad \text{(A78)}$$

Note that (A65) holds if (A77) holds.

Proof of Lemma 9

Combining (29) with (34) and (33), we arrive at

$$\Omega_v(t) \leq \rho_4(\beta_u(\rho_3(\Omega_v(0)), 0)) \quad \text{(A79)}$$

where Ω_v is defined in (35). Hence, for all the initial conditions of the closed-loop system (10)–(12), (A3), and (22) that satisfy

$$|X(0)| + \|v(0)\|_\infty + \|v_z(0)\|_\infty < \delta_v \quad \text{(A80)}$$

with

$$\delta_v \leq \bar{\gamma}^{-1}(\delta_1) \quad \text{(A81)}$$

where $\bar{\gamma}$ is the following class $\mathcal{K}$ function:

$$\bar{\gamma}(s) = \rho_4(\beta_u(\rho_3(s,0))). \quad \text{(A82)}$$

The solutions of the system satisfy (38), and hence, satisfy (13)–(15). From Lemma 6, the lemma is proved by choosing

$$\delta_u \leq \rho_5^{-1}(\delta_v). \quad \text{(A83)}$$

References

[1] D. Angeli and E. D. Sontag, "Forward completeness, unboundedness observability, and their Lyapunov characterizations," *Syst. Control Lett.*, vol. 38, pp. 209–217, 1999.

[2] Z. Artstein, "Linear systems with delayed controls: A reduction," *IEEE Trans. Automat. Control*, vol. AC-27, no. 4, pp. 869–879, Aug. 1982.

[3] N. Bekiaris-Liberis and M. Krstic, "Compensation of time-varying input and state delays for nonlinear systems," *J. Dyn. Syst. Meas. Control*, vol. 134, 2012, Paper 011009.

[4] N. Bekiaris-Liberis and M. Krstic, "Compensation of state-dependent input delay for nonlinear systems," *IEEE Trans. Automat. Control*, vol. 58, no. 2, pp. 275–289, Feb. 2013.

[5] N. Bekiaris-Liberis and M. Krstic, *Nonlinear Control Under Nonconstant Delays*. Philadelphia, PA, USA: SIAM, 2013.

[6] N. Bekiaris-Liberis and M. Krstic, "Robustness of nonlinear predictor feedback laws to time- and state-dependent delay perturbations," *Automatica*, vol. 49, pp. 1576–1590, 2013.

[7] N. Bekiaris-Liberis and M. Krstic, "Nonlinear control under delays that depend on delayed states," *Eur. J. Control*, vol. 19, pp. 389–398, 2013.

[8] D. Bresch-Pietri, J. Chauvin, and N. Petit, "Prediction-based feedback control of a class of processes with input-varying delay," in *Proc. Amer. Control Conf.*, 2012, pp. 1991–1997.

[9] D. Bresch-Pietri, J. Chauvin, and N. Petit, "Invoking Halanay inequality to conclude on closed-loop stability of processes with input-varying delay," in *Proc. IFAC Workshop Time Delay Syst.*, 2012, pp. 266–271.

[10] D. Bresch-Pietri, J. Chauvin, and N. Petit, "Sufficient conditions for the prediction-based stabilization of linear systems subject to input with input-varying delay," in *Proc. Amer. Control Conf.*, 2013, pp. 144–151.

[11] D. Bresch-Pietri, J. Chauvin, and N. Petit, "Prediction-based stabilization of linear systems subject to input-dependent input delay of integral type," *IEEE Trans. Automat. Control*, vol. 59, no. 9, pp. 2385–2399, Sep. 2014.

[12] D. Bresch-Pietri and N. Petit, "Robust compensation of a chattering time-varying input delay," in *Proc. IEEE Conf. Decis. Control*, Los Angeles, CA, USA, 2014, pp. 457–462.

[13] X. Cai and M. Krstic, "Nonlinear control under wave actuator dynamics with time-and state-dependent moving boundary," *Int. J. Robust Nonlinear Control*, vol 25, pp. 222–251, 2015.

[14] X. Cai and M. Krstic, "Nonlinear stabilization through wave PDE dynamics with a moving uncontrolled boundary," *Automatica*, vol. 68, pp. 27–38, 2016.

[15] M. Chebre, Y. Creff, and N. Petit, "Feedback control and optimization for the production of commercial fuels by blending," *J. Process Control*, vol. 20, pp. 441–451, 2010.

[16] C.-H. Clerget, J.-P. Grimaldi, M. Chebre, and N. Petit, "Optimization of dynamical systems with time-varying or input-varying delays," in *Proc. IEEE Conf. Decis. Control*, Las Vegas, NV, USA, 2016, pp. 2282–2289..

[17] C.-H. Clerget, N. Petit, and L. T. Biegler, "Dynamic optimization of a system with input-dependent time delays," in *Proc. Found. Comput. Aided Process Oper./Chem. Process Control*, Tucson, AZ, USA, 2017.

[18] J.-M. Coron, *Control and Nonlinearity*. Providence, RI, USA: Amer. Math. Soc., 2007.

[19] C. Depcik and D. Assanis, "One-dimensional automotive catalyst modeling," *Prog. Energy Combustion Sci.*, vol. 31, pp. 308–369, 2005.

[20] M. Diagne, N. Bekiaris-Liberis, and M. Krstic, "Time- and state-dependent input delay-compensated Bang-Bang control of a screw extruder for 3D printing," *Int. J. Robust Nonlinear Control*, vol. 27, no. 17, pp. 3727–3757, Nov. 2017.

[21] M. Diagne, N. Bekiaris-Liberis, and M. Krstic, "Compensation of input delay that depends on delayed input," in *Proc. Amer. Control Conf.*, Seattle, WA, USA, 2017, pp. 5153–5158.

[22] M. Diagne and M. Krstic, "State-dependent input delay-compensated bang-bang control: Application to 3D printing based on screw extruder," in *Proc. Amer. Control Conf.*, Chicago, IL, USA, 2015, pp. 5653–5658.

[23] M. Hansen, “Model based control of single-phase marine cooling systems,” Ph.D. thesis, Dept. Electron. Syst., Aalborg Univ., Aalborg, Denmark, 2013.

[24] J. Harmand and D. Dochain, “The optimal design of two interconnected (bio) chemical reactors revisited,” *Comput. Chem. Eng.*, vol. 30, pp. 70–82, 2005.

[25] J. P. Hathout, M. Fleifil, A. M. Annaswamy, and A. F. Ghoniem, “Active control using fuel-injection of time-delay induced combustion instability,” *AIAA J. Propulsion Power*, vol. 18, pp. 390–399, 2002.

[26] M. Herty, J.-P. Lebacque, and S. Moutari, “A novel model for intersections of vehicular traffic flow,” *Netw. Heterogenous Media*, vol. 4, pp. 813–826, 2009.

[27] I. Karafyllis and M. Krstic, *Predictor Feedback for Delay Systems: Implementations and Approximations*. Basel, Switzerland: Birkhäuser, 2017.

[28] H. Khalil, *Nonlinear Systems*, 3rd ed. Englewood Cliffs, NJ, USA: Prentice-Hall, 2002.

[29] M. Krstic, *Delay Compensation for Nonlinear, Adaptive, and PDE Systems*. Basel, Switzerland: Birkhäuser, 2009.

[30] M. Krstic, “Input delay compensation for forward complete and feedforward nonlinear systems,” *IEEE Trans. Automat. Control*, vol. 55, no. 2, pp. 287–303, Feb. 2010.

[31] F. Mazenc and M. Malisoff, “Reduction model approach for systems with a time-varying delay,” *Proc. IEEE Conf. Decis. Control*, Osaka, Japan, 2015, pp. 7723–7727.

[32] F. Mazenc and M. Malisoff, “New prediction approach for stabilizing time-varying systems under time-varying input delay,” *IEEE Conf. Decis. Control*, Las Vegas, NV, USA, 2016, pp. 3178–3182.

[33] F. Mazenc, M. Malisoff, and S.-I. Niculescu, “Stability and control design for time-varying systems with time-varying delays using a trajectory based approach,” *SIAM J. Control Optim.*, vol. 55, no. 1, pp. 533–556, 2017.

[34] S. Mondie and W. Michiels, “Finite spectrum assignment of unstable time-delay systems with a safe implementation,” *IEEE Trans. Automat. Control*, vol. 48, no. 12, pp. 2207–2212, Dec. 2003.

[35] N. Petit, Y. Creff and P. Rouchon, “Motion planning for two classes of nonlinear systems with delays depending on the control,” in *Proc. IEEE Conf. Decis. Control*, Tampa, FL, USA, 1998, vol. 1, pp. 1007–1011.

[36] J.-P. Richard, “Time-delay systems: An overview of some recent advances and open problems,” *Automatica*, vol. 39, pp. 1667–1694, 2003.

[37] L. Roca, M. Berenguel, L. Yebra, and D. C. Alarcon-Padilla, “Solar field control for desalination plants,” *Solar Energy*, vol. 82, pp. 772–786, 2008.

[38] M. Sbarciog, R. De Keyser, S. Cristea, and C. De Prada, “Nonlinear predictive control of processes with variable time delay. A temperature control case study,” in *Proc. IEEE Int. Conf. Control Appl.*, San Antonio, TX, USA, 2008, pp. 1001–1006.

[39] E. Sontag and Y. Wang, “On characterizations of the input-to-state stability property,” *Syst. Control Lett.*, vol. 24, pp. 351–359, 1995.

[40] K. Zenger and A. J. Niemi, “Modeling and control of a class of time-varying continuous flow processes,” *J. Process Control*, vol. 19, pp. 1511–1518, 2009.

[41] Q.-C. Zhong, “On distributed delay in linear control laws—Part I: Discrete-delay implementations,” *IEEE Trans. Automat. Control*, vol. 49, no. 11, pp. 2074–2080, Nov. 2004.