Predictor-Based Adaptive Cruise Control Design

Nikolaos Bekiaris-Liberis, Claudio Roncoli, and Markos Papageorgiou, *Fellow, IEEE*

***Abstract*— We develop a predictor-based adaptive cruise control design with integral action (based on a nominal constant time-headway policy) for the compensation of large actuator and sensor delays in vehicular systems utilizing measurements of the relative spacing as well as of the speed and the short-term history of the desired acceleration of the ego vehicle. By employing an input–output approach, we show that the predictor-based adaptive cruise control law with integral action guarantees all of the four typical performance specifications of adaptive cruise control designs, namely, 1) stability, 2) zero steady-state spacing error, 3) string stability, and 4) non-negative impulse response, despite the large input delay. The effectiveness of the developed control design is shown in simulation considering various performance metrics.**

***Index Terms*— Predictor feedback, delay systems, adaptive cruise control, string stability.**

I. Introduction

A. Motivation

ACTUATOR and sensor delays are ubiquitous in vehicles equipped with Adaptive Cruise Control (ACC) systems. Among other reasons, actuator delays may be due to engine response, throttle or brake actuators, and computational time, whereas sensor delays may be due to radar or lidar systems, wheel speed sensors, and sampling of measurements [7], [10], [11], [27], [35], [36], [50], [52], [54], [55].

The presence of such delays deteriorates the performance of ACC algorithms when these algorithms are designed ignoring the presence of the delay. Among the most severe consequences for the emerging traffic flow are the decrease in traffic capacity, the loss of string stability, and even the loss of individual vehicle stability. As a matter of fact, a decrease in capacity implies reduced traffic throughput and increased congestion, whereas the degradation of the stability or string stability properties imply reduced comfort and safety, and increased fuel consumption [7], [8], [10], [20], [27], [33], [35], [43], [50], [52]–[55].

B. Literature

Despite the significant need for delay compensation in ACC-equipped vehicles, the vast majority of existing ACC strategies does not take into account the effect of such delays [11]–[13], [20], [21], [24], [36], [37], [39]–[42], [44], [46], [47], [49], [58]. However, robustness analysis tools of various ACC strategies to delays are developed [7], [10], [43], [52], [55], which reveal the need of restricting substantially the delay value in order to guarantee string or even vehicle stability.

Exceptions to this rule are the papers [50], [53], and [54]. In the first two papers a discrete-time version of a predictor-based strategy is presented, whereas in the third paper a Model Predictive Control-based (MPC-based) delay-compensating strategy is developed. Yet, none of these papers proves string stability or stability of each individual vehicular system (based on the original, continuous-time system). In addition, no formal connection is made with the classical predictor-based control design methodology developed in the late 1970s [2], [5], [14]–[16], [18], [22], [23], [28], [31], [57], which is made in the present paper and which offers an opportunity of exploiting this control design methodology for ACC design. Finally, none of the mentioned papers is addressing the problem of the simultaneous compensation of both actuator and sensor delays.

Although such systems are not considered in the present paper, it is worth to mention that Cooperative Adaptive Cruise Control (CACC) systems may also have delay-compensating capabilities, see, for example, [4], [9], [25], [30], [32], [34], [38], [42], [46], [49], [56]. This may be attributed, for instance, to the fact that vehicles may exchange information about their desired acceleration through vehicle-to-vehicle (V2V) communication.

C. Contributions

In this paper, utilizing a constant time-headway nominal ACC design, the predictor-based feedback design methodology is employed for compensation of long actuator and sensor delays in vehicular systems modeled or approximated by a second-order linear system. Measurements of the relative spacing as well as the speed and the history, over a window equal to the delay length, of the control input (desired acceleration) of each individual vehicular system are utilized to compute the control input for each vehicle. Employing an input-output approach, we prove that the predictor-based ACC law with integral action guarantees all four typical requirements of ACC designs, see, e.g., [12], namely, (1) stability of each individual vehicular system, (2) zero steady-state spacing error between the actual and the desired inter-vehicle spacing, (3) string stability of homogenous platoons of vehicular systems, and (4) non-negative impulse response of each individual vehicular system, for any delay value smaller than the desired time-headway, which constitutes a physically intuitive limitation.

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N. Bekiaris-Liberis and M. Papageorgiou are with the Department of Production Engineering and Management, Technical University of Crete, 73100 Chania, Greece (e-mail: nikos.bekiaris@dssl.tuc.gr; markos@dssl.tuc.gr).

C. Roncoli is with the Department of Built Environment, School of Engineering, Aalto University, 02150 Espoo, Finland (e-mail: claudio.roncoli@aalto.fi).

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Note that analogous ACC designs without delay compensation require that the delay value is smaller than half the time-headway, e.g., [52], [55]. Note also that the integral term in the proposed predictor-based ACC design is needed to ensure requirement (2), which may not be fulfilled in the absence of this term. We also establish string stability robustness of the predictor-based ACC law to delay mismatch between the real delay and the delay value available to the designer.

The performance of the developed ACC algorithm is verified in simulation and compared with an existing ACC strategy considering seven different performance indices that provide quantitative performance measures for four common physical requirements of ACC designs, namely, (1) tracking error, (2) safety, (3) fuel consumption, and (4) comfort.

In order to help the reader to better understand the key conceptual ideas as well as the technical intricacies of the predictor-based ACC design methodology, we adopt a rather pedagogical exposition approach by first presenting the predictor-based ACC design without the integral term.

D. Organization

In Section II we introduce the predictor-based ACC strategy without integral action and in Section III we study its stability and string stability properties. The effectiveness of the predictor-based ACC strategy without integral action is illustrated via a numerical example in Section IV. In Section V the predictor-based ACC design is extended to incorporate an integral term; moreover, the stability as well as string stability analyses of individual vehicular systems and platoons of vehicular systems, respectively, under the new design are presented. String stability robustness of the predictor-based feedback control law to delay mismatch is established in Section VI. The performance of the ACC law with integral action is validated in simulation and compared to an alternative ACC strategy employing four different performance indices in Section VII. Finally, we provide further issues of our current research and discuss possible future directions in Section VIII.

E. Notation

For a complex number s we denote by $|s|$ its absolute value. The Laplace transform of a function $f(t)$, $t \geq 0$, is denoted by $F(s) = \mathcal{L}\{f(t)\}$. The temporal norm $\mathscr{L}_p$, $p \in [1,\infty]$, of a signal $f(t)$, $t \geq 0$, is defined as

$$\|f\|_p = \begin{cases} \left(\int_0^\infty |f(t)|^p dt\right)^{\frac{1}{p}}, & p \in [1,\infty) \\ \sup_{t\geq 0} |f(t)|, & p = \infty \end{cases}. \tag{1}$$

We denote by $\mathscr{L}_p$ the space of signals with bounded $\mathscr{L}_p$ norm.

F. Definitions

We adopt the classical definition of stability, see, e.g., [19]. Furthermore, we adopt the definition of string stability from [6], which is an adaptation of the original definition of string stability for general interconnected nonlinear systems from [44] to the case of interconnected systems of vehicles following each other in a single lane. We say that an interconnected system of vehicles, indexed by $i = 1,\ldots,N$, where $i = 1$ denotes the first vehicle, following each other in a single lane without overtaking, is string stable when the following hold

$$\|\delta_i\|_p \leq \|\delta_{i-1}\|_p \tag{2}$$
$$\|v_{\mathrm{r}_i}\|_p \leq \|v_{\mathrm{r}_{i-1}}\|_p, \quad \forall\ p \in [1,\infty] \text{ and } i = 2,\ldots,N \tag{3}$$
$$\|a_{\mathrm{r}_i}\|_p \leq \|a_{\mathrm{r}_{i-1}}\|_p, \tag{4}$$

where

$$\delta_i = s_i - hv_i, \tag{5}$$

with the spacing $s_i = y_{i-1} - y_i - l_i$, $i = 1,\ldots,N$, with y_j being the position of vehicle j and l_i being its length; v_i denotes the speed of vehicle i, $h > 0$ is the desired constant time-headway, and

$$v_{\mathrm{r}_i} = v_{i-1} - v_i \tag{6}$$
$$a_{\mathrm{r}_i} = a_{i-1} - a_i, \tag{7}$$

where a_i denotes the acceleration of vehicle i. Note that we adopt the convention that $v_0 = v_{\mathrm{L}}$ and $a_0 = a_{\mathrm{L}}$, where v_{L} and a_{L} are the speed and acceleration of the string leader, respectively (see Fig. 1).

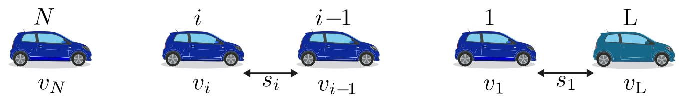

Fig. 1. Platoon of $N+1$ vehicles following each other in a single lane without overtaking. The dynamics of each vehicle $i = 1,\ldots,N$ are governed by system (8), (9). Each vehicle can measure its own speed and the spacing with respect to the preceding vehicle. The dynamics of the leading vehicle satisfy $\ddot{y}_{\mathrm{L}} = a_{\mathrm{L}}$, where y_{L} and a_{L} are the position and acceleration of the leading vehicle, respectively.

II. Predictor-Based Control of ACC-Equipped Vehicles With Actuator Delay

A. Vehicle Dynamics

We consider a homogenous string of autonomous vehicles (see Fig. 1) each one modeled by the following second-order linear system with input delay, see, e.g., [10], [13], [35], [43], [50], [55]

$$\dot{s}_i(t) = v_{i-1}(t) - v_i(t) \tag{8}$$
$$\dot{v}_i(t) = u_i(t - D), \tag{9}$$

$i = 1,\ldots,N$, where s_i and v_i are defined in Section I-F, u_i is the individual vehicle's control variable, $D > 0$ is actuator delay, and $t \geq 0$ is time. Note that a uniform equilibrium point of system (8), (9) for all vehicles is obtained when all vehicles have zero acceleration and their speed is dictated by the speed of the leader. System (8), (9) may come from linearization of a nonlinear model around a uniform (for all vehicles) operating point, and thus, the states s_i and v_i may represent the error between the actual spacing and speed from some nominal constant spacing and speed, respectively.

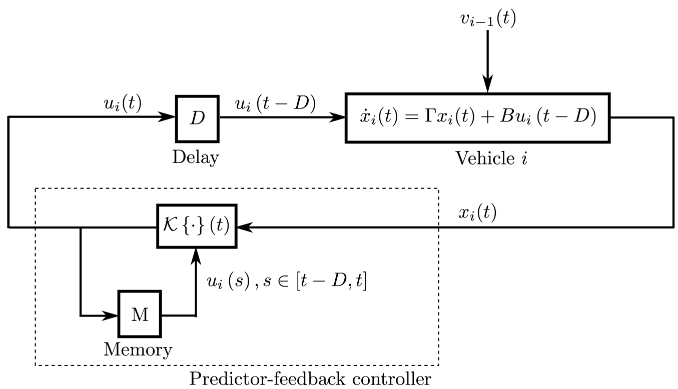

Fig. 2. Block diagram of the predictor-feedback control design (12). The operator $\mathcal{K}\{\cdot\}$ is defined as $\mathcal{K}\{x,u\}(t) = K\left(e^{\Gamma D} x_i(t) + \int_{t-D}^{t} e^{\Gamma(t-\theta)} B u_i(\theta) d\theta\right)$.

B. Delay-Free Control Design

In the absence of the actuator delay D, the following constant time-headway control strategy is widely used, either as it is or as a special case of more general control designs, see, e.g., [7], [10], [13]:

$$u_i(t) = \alpha\left(\frac{s_i(t)}{h} - v_i(t)\right), \tag{10}$$

where α and h are positive design parameters that represent control gain and desired time-headway, respectively. Using the nominal transfer function

$$\begin{aligned} G_{\text{nom}}(s) &= \frac{V_i(s)}{V_{i-1}(s)}, i = 1, \ldots, N \\ &= \frac{\frac{\alpha}{h}}{s^2 + \alpha s + \frac{\alpha}{h}}, \end{aligned} \tag{11}$$

it can be shown that a homogenous platoon of vehicles with dynamics (8), (9) under the control law (10) with $\alpha \geq \frac{4}{h}$, is stable and string stable in the $\mathcal{L}_p$, $p \in [1, \infty]$, sense, see, e.g., [6], [10].

Remark 1: In the case of a homogenous platoon, stability and string stability may both be studied merely on the basis of a single transfer function, namely, transfer function $G(s) = \frac{V_i(s)}{V_{i-1}(s)}$, $i = 1, \ldots, N$. This holds true because all transfer functions that may relate either the spacing errors, or the speed, or the acceleration, or the relative speed and acceleration errors (see relations (6) and (7), respectively), between two consecutive vehicles, are identical to each other (see, e.g., [6], [24]).

C. Predictor-Based Control Design

The predictor-based control laws for system (8), (9) are given by (see Fig. 2)

$$u_i(t) = K\left(e^{\Gamma D} x_i(t) + \int_{t-D}^{t} e^{\Gamma(t-\theta)} B u_i(\theta) d\theta\right), \tag{12}$$

where

$$\Gamma = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \tag{13}$$

$$K = \begin{bmatrix} \frac{\alpha}{h} & -\alpha \end{bmatrix} \tag{14}$$

$$x_i = \begin{bmatrix} s_i \\ v_i \end{bmatrix} \tag{15}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{16}$$

One should notice that the control law (12) is suitable for autonomous operation since it employs only measurements of the current spacing s_i and speed v_i, as well as of the past D-second history of the control variable u_i, which are available to vehicle i using on-board sensors, see, e.g., [11], [12], [24], [35], [36], [45], [50], [52], [54], [58]. Note also that in the absence of the delay, i.e., when $D = 0$, the control law (12) reduces to the nominal, delay-free control design (10). The control law (12) was developed in [2] and [28]; not only its stability and robustness properties are extensively studied in the literature [5], [14], [18], [23], but, in addition, several implementation methodologies were developed [18], [31].

For the readers' convenience the basic principles of predictor feedback are reviewed in Appendix A.

We analyze next, adopting a transfer function approach, the stability and string stability properties of a homogenous platoon of vehicles modeled by system (8), (9) under the ACC law (12).

III. Stability and String Stability Analysis Under Predictor-Based Feedback for Homogenous Platoons

Theorem 1: Consider a homogenous platoon of vehicles with dynamics modeled by system (8), (9) under the control laws (12). Then, each individual vehicular system is stable. If, in addition, $\alpha \geq \frac{4}{h}$, then the platoon is string stable in the $\mathcal{L}_p$, $p \in [1, \infty]$, sense, for any $D \geq 0$.

Proof: See Appendix B. ■

Remark 2: One best appreciates the stability and string stability results of Theorem 1 by considering the fact that no restriction on the magnitude of the delay is imposed (which is inherent to the nature of such predictor-based control laws since the delay is completely compensated, see, e.g., [5], [23]), in contrast to the case of the uncompensated control law, which requires $h \geq 2D$ for a choice of α and h to exist such that the system is both stable and string stable, see, e.g., [10], [55]. In fact, the condition $h \geq 2D$ is necessary also in the case where one employs an extra term of the form $b(v_{i-1} - v_i)$ in the nominal feedback law (10), see, e.g., [10], [55].

Moreover, in the case of the uncompensated control law, the resulting transfer function is given by

$$G(s) = \frac{e^{-Ds}\frac{\alpha}{h}}{s^2 + \alpha s e^{-Ds} + \frac{\alpha}{h} e^{-Ds}}. \tag{17}$$

Although the analytical study of string stability based on (17) is performed, for instance, in [10] and [25], it is very difficult to analytically study $\mathcal{L}_p$, $p \in [1, \infty]$, string stability using (17). In contrast, due to the fact that the denominator of the respective transfer function under predictor feedback is a second-order polynomial in s, identical to the denominator of (11) (see relation (B.12)), $\mathscr{L}_p$, $p \in [1, \infty]$, string stability can be established much more easily.

Remark 3: Note that in the case of sensor delay, i.e., when a measurement of $x_i(t-D)$ is available, and there is no actuator delay, one could employ the following control law, see, e.g., [23], [51]

$$u_i(t) = K\left(e^{\Gamma D} x_i(t-D) + \int_{t-D}^{t} e^{\Gamma(t-\theta)} B u_i(\theta) d\theta\right). \quad (18)$$

Repeating the computations in the proof of Theorem 1 it can be shown that the resulting transfer function $G(s) = \frac{V_i(s)}{V_{i-1}(s)}$ is identical to the one obtained in the case of actuator delay (see relation (B.12)), and thus, the same stability (see also [23], [51]) and string stability results hold in this case as well. In the case where there are both input and sensor delays, say, D and D_s, respectively, the control law can be modified to, see, e.g., [23]

$$u_i(t) = K\left(e^{\Gamma(D+D_s)} x_i(t-D_s) + \int_{t-D-D_s}^{t} e^{\Gamma(t-\theta)} B u_i(\theta) d\theta\right). \quad (19)$$

The resulting transfer function $G(s) = \frac{V_i(s)}{V_{i-1}(s)}$ is then given by

$$G(s) = \frac{e^{-(D+D_s)s}\frac{\alpha}{h}}{s^2 + \alpha s + \frac{\alpha}{h}}. \quad (20)$$

Stability (see also [23], [51]) and string stability follow by Theorem 1.

Remark 4: Note that the steady-state spacing error of the first vehicle in the string, which follows the leader, under the delay-compensating control law (12) is not zero. One can see this by deriving the transfer function $\frac{\Delta_1(s)}{Y_L(s)}$, which satisfies

$$\frac{\Delta_1(s)}{Y_L(s)} = 1 - (1+sh)\, G(s) = \frac{s^2 + \alpha s\left(1-e^{-sD}\right) + \frac{\alpha}{h}\left(1-e^{-sD}\right)}{s^2 + \alpha s + \frac{\alpha}{h}}, \quad (21)$$

and which is different than the respective transfer function $G(s) = \frac{\Delta_i(s)}{\Delta_{i-1}(s)}$, $i = 2, \ldots, N$, for the rest of the vehicles (see equation (B.12)). Since each vehicular system is stable, for a constant steady-state speed of the leader, say equal to v_{ss}, which implies that $Y_L(s) = \frac{v_{ss}}{s^2}$, the steady-state spacing error is given by the final value theorem as

$$\begin{aligned}\delta_{1ss} &= \lim_{s\to 0} s\,(1-(1+sh)\,G(s))\,\frac{v_{ss}}{s^2} \\ &= v_{ss}\lim_{s\to 0}\frac{s^2 + \alpha s\left(1-e^{-sD}\right) + \frac{\alpha}{h}\left(1-e^{-sD}\right)}{s\left(s^2+\alpha s+\frac{\alpha}{h}\right)} \\ &= D v_{ss} \quad (22)\\ &\neq 0, \quad (23)\end{aligned}$$

where we used the fact that $\lim_{s\to 0}\frac{s^2+\alpha s\left(1-e^{-sD}\right)+\frac{\alpha}{h}\left(1-e^{-sD}\right)}{s} = \frac{D\alpha}{h}$.* This is in accordance to the result in [13] in which the disturbance attenuation limitations of systems with input delays, under any time-invariant feedback controller, are provided.

*Another way to see that the final value theorem can be applied is by noting that $\frac{D\alpha}{h} < \infty$, which implies that $s = 0$ is not a pole of the function $s\Delta_1(s)$, and thus, all poles of $s\Delta_1(s)$ are on the left-hand complex plane.

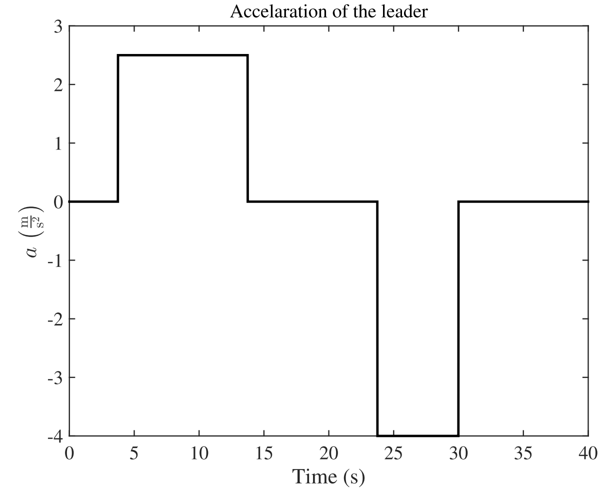

Fig. 3. Acceleration maneuver of the leader.

IV. Simulation

We present a simulation study considering a homogenous platoon of four vehicles with dynamics given by (8), (9) following a leader with dynamics defined as

$$\dot{y}_L(t) = v_L(t) \quad (24)$$
$$\dot{v}_L(t) = a_L(t), \quad (25)$$

where y_L and v_L are the position and speed of the leading vehicle, respectively, and a_L is the leader's acceleration, which is regarded as a reference input chosen as the step input signal shown in Fig. 3. Note that the AASHTO standard for comfortable deceleration is $-3.4\ \frac{m}{s^2}$, and thus, according to Fig. 3 the deceleration of vehicles may exceed this threshold. Yet, this is because the vehicles are capable of tracking the reference deceleration of the leader and not because the responses of their decelerations, to the leader's maneuver, exhibit an overshoot. We choose such a small value for the minimum of the reference deceleration profile that the vehicles should track to highlight that the developed ACC law is able to successfully handle such extreme situations.

We choose the desired time-headway as $h = \frac{2}{\pi}$ s and the delay as $D = 0.4$ s. We compare the response of the string of the four vehicles to a step acceleration signal a_L to the cases where the delay-uncompensated strategy (Fig. 4)

$$u_i(t) = \frac{\alpha}{h} s_i(t) - \alpha v_i(t) + b\left(v_{i-1}(t) - v_i(t)\right), \quad (26)$$

with $\alpha = 1$, $b = 0.8$, see, e.g., [10], and the delay-compensating strategy (12) with $\alpha = 2\pi$ (Fig. 5) are employed. Note that there exists no choice of (α, b) in the uncompensated strategy (26) that guarantees both stability and string stability for these values of D and h as it is shown in [55]. However, with the choice $\alpha = 1$, $b = 0.8$, each individual vehicular system is stable [55]. In contrast, the delay-compensating strategy achieves both stability and $\mathscr{L}_p$,

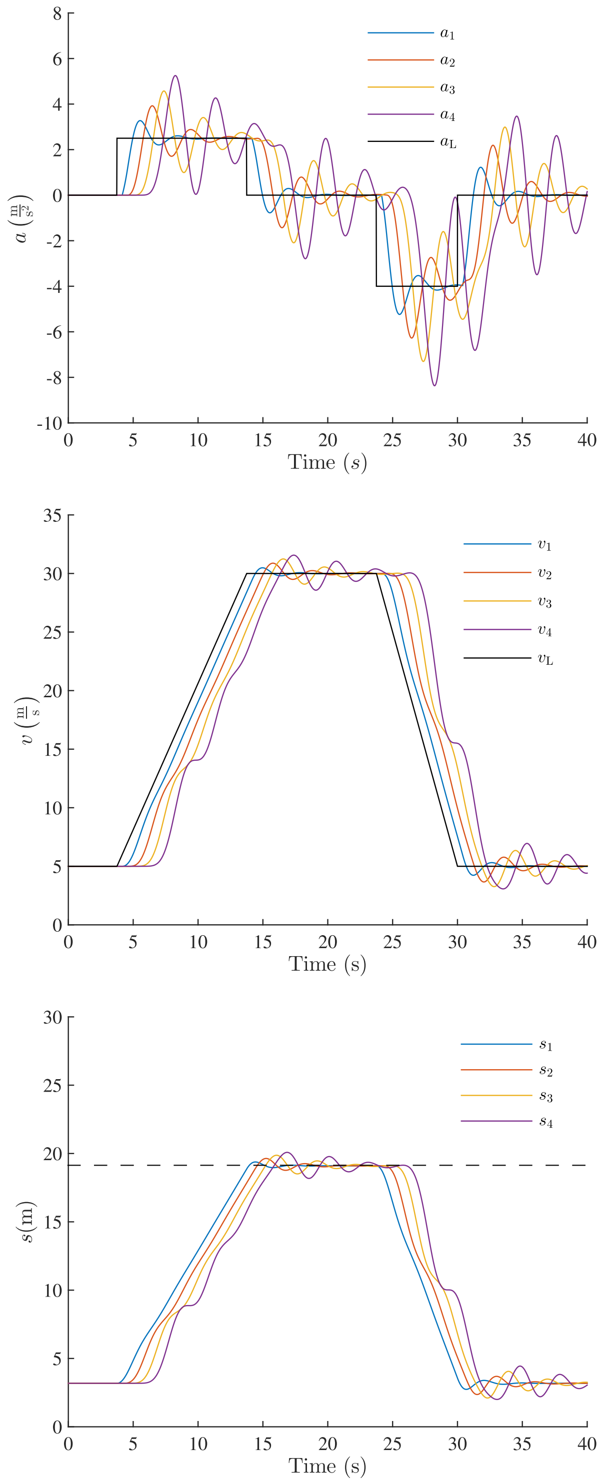

Fig. 4. Acceleration (top), speed (middle), and spacing (bottom) of four vehicles following a leader that performs the acceleration maneuver shown in Fig. 3, under the nominal, uncompensated ACC strategy (26). The reference spacing $\frac{2}{\pi} \times 30 \approx 19.1$ m is depicted with dashed line.

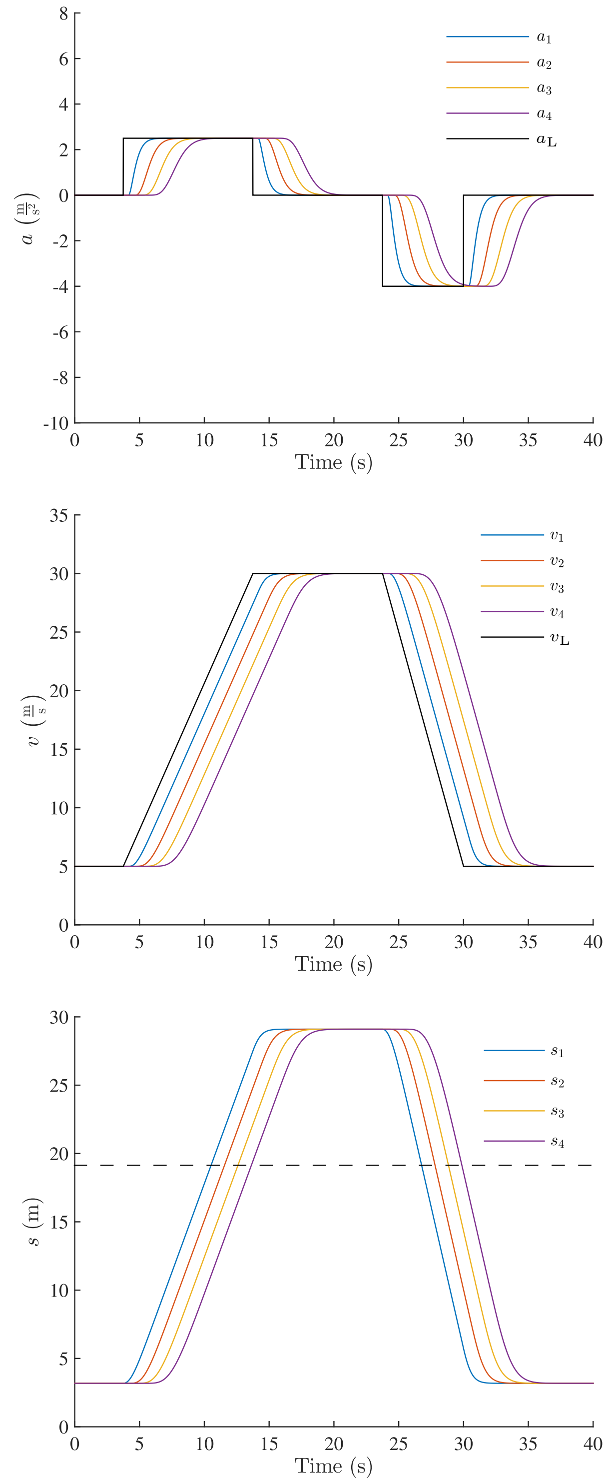

Fig. 5. Acceleration (top), speed (middle), and spacing (bottom) of four vehicles following a leader that performs the acceleration maneuver shown in Fig. 3, under the delay-compensating ACC strategy (12). The reference spacing $\frac{2}{\pi} \times 30 \approx 19.1$ m is depicted with dashed line.

$p \in [1, \infty]$, string stability since the condition $\alpha \geq \frac{4}{h}$ is satisfied. Note that, as explained in Remark 4, the delay-compensating strategy does not guarantee that the steady-state spacing error is zero as shown in Fig. 5 (the desired steady-state spacing is $\frac{2}{\pi} \times 30 \approx 19.1$ m).

The particular value for the time headway is taken from [10], [56] and the motivation for this choice is explained as follows (see [10], [56]). In the numerical investigations in [10] and [56], the time headway is chosen such that, when the traffic flow is at equilibrium, the maximum of the derivative of the so-called "desired speed" or "range policy" of vehicles is achieved, which physically corresponds to a *worst-case* scenario for the traffic system in terms of both individual vehicle's stability as well as string stability, see, [10], [56]. Motivated by this fact, utilizing numerical values that correspond to realistic traffic data the particular numerical value $\frac{2}{\pi}$ for the time headway is obtained in [10] and [56]. For this reason, as well as for the sake of fair comparison between the results in those papers and the present paper, we also choose this particular numerical value for the time headway.

The particular value of the delay is also taken from [10] and [56] and its choice is motivated as follows. The value for the ratio between the time headway and the delay is chosen to highlight the fact that with the proposed predictor-feedback design one doesn't need to restrict the delay to be at most half the time headway (i.e., about 0.3). In addition, this particular value for the delay is sufficiently large so it could be considered as representative of a "total" delay, which may appear due to several reasons, such as, for example, computation or sensing times, see, e.g., [10], [56]. Finally, since this delay value is used in some of the numerical investigations in [10] and [56] for studying the stability and string stability properties of the considered systems, for a fair comparison between the results in the present paper and the results in [10] and [56] in terms of the achievable performance, we use this particular delay value as well.

We demonstrate next the stability and $\mathcal{L}_2$ string stability (which can be proved specializing Theorem 3 in Section VI to the case without integral action) robustness properties of the predictor-based ACC design to delay mismatch. In Fig. 6 we show the response of the system when the delay value that is known to the designer is $D = 0.35$, whereas the real delay value is $D_{\mathrm{r}} = 0.4$, that is, a 12.5% uncertainty in the delay value is introduced. One can observe that stability and $\mathcal{L}_2$ string stability are preserved despite the uncertainty in the delay. The $\mathcal{L}_2$ string stability robustness of the predictor-based ACC design is also illustrated in Fig. 7, which shows $|G_1(j\omega)|$, where $G_1(s) = \frac{V_i(s)}{V_{i-1}(s)}$, $i = 1, \ldots, N$, is defined for the case in which there is uncertainty in the value of the actual delay (see relation (F.1) for the special case without integral action), for four different values of the delay that is available to the designer. One can observe that $|G_1(j\omega)|$ never exceeds unity for any delay value D that satisfies $0.31 \leq D \leq 0.49$, i.e., for any absolute error smaller than 0.08, whereas for any $D \in [0.31, 0.49]$ the corresponding (more conservative) absolute error obtained within the analytical proof of Theorem 3 (for the special case without integral action) is 0.012. Note that it is verified in simulation that stability is also preserved for any $D \in [0.31, 0.49]$.

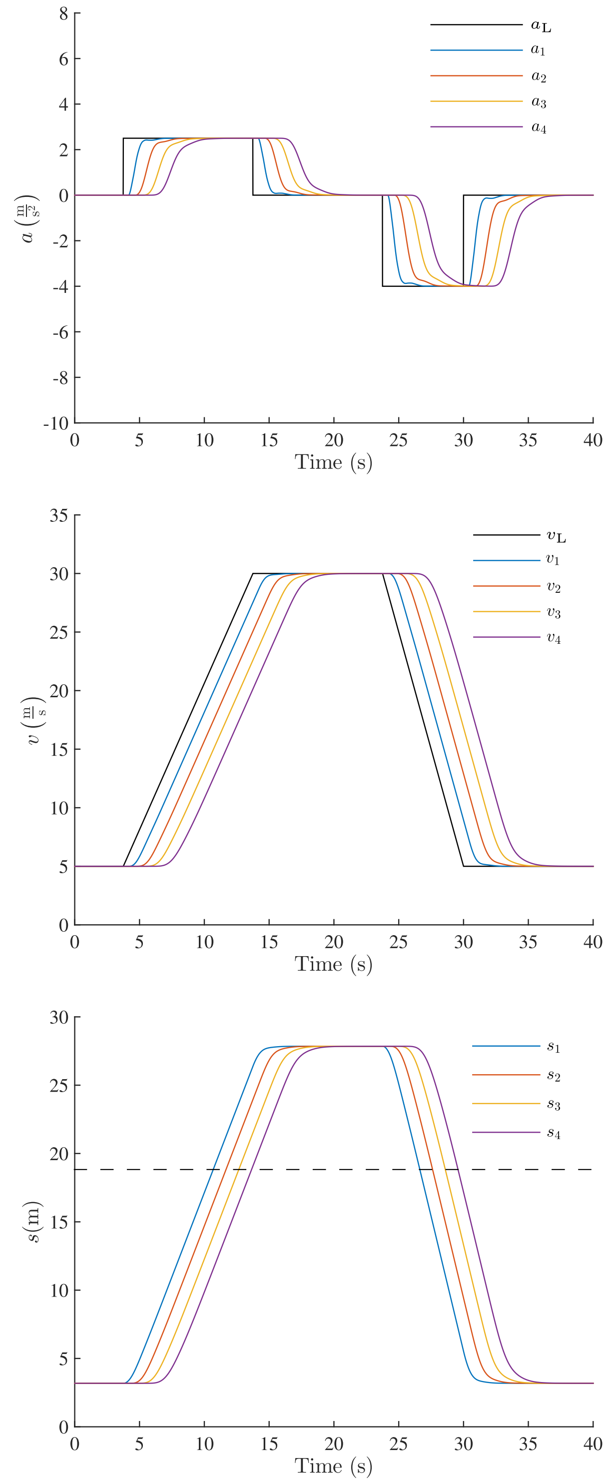

Fig. 6. Acceleration (top), speed (middle), and spacing (bottom) of four vehicles following a leader that performs the acceleration maneuver shown in Fig. 3, under the delay-compensating ACC strategy (12) and 12.5% uncertainty in the delay value. The reference spacing $\frac{2}{\pi} \times 30 \approx 19.1$ m is depicted with dashed line.

V. Predictor-Based ACC With Integral Action

The predictor-based ACC law developed in Section II is proportional in the sense that the nominal, i.e., for the delay-free case, ACC design (10) is a proportional controller for the spacing error (5). We now augment this proportional control law to incorporate an integral action for the spacing error (5) in order to eliminate the steady-state spacing error of the previous design. Defining the state of the i-th integrator as

$$\dot{\sigma}_i(t) = \frac{1}{h} s_i(t) - v_i(t), i = 1, \ldots, N, \tag{27}$$

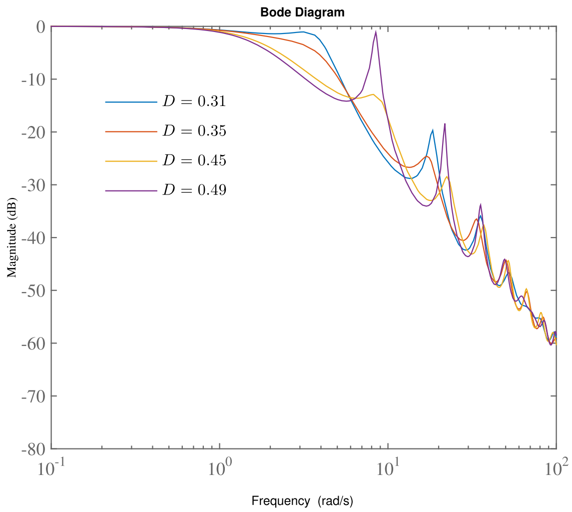

Fig. 7. Bode diagram of the magnitude of the transfer function $G_1(s) = \frac{V_i(s)}{V_{i-1}(s)}$, $i = 1, \ldots, N$, (see relation (F.1) for the special case without integral action), when there is uncertainty in the value of the real delay $D_{\rm r}$, for four different values of the delay D that is available to the designer.

the ACC laws with integral action are given for all $i = 1, \ldots, N$ by

$$u_i(t) = \bar{K}\left(e^{\bar{\Gamma}D}\bar{x}_i(t) + \int_{t-D}^{t} e^{\bar{\Gamma}(t-\theta)}\bar{B}u_i(\theta)d\theta\right), \quad (28)$$

where

$$\bar{\Gamma} = \begin{bmatrix} 0 & 0 & -1 \\ \frac{1}{h} & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad (29)$$

$$\bar{K} = \begin{bmatrix} \bar{k}_1 & \bar{k}_2 & \bar{k}_3 \end{bmatrix} \quad (30)$$

$$\bar{x}_i = \begin{bmatrix} s_i \\ \sigma_i \\ v_i \end{bmatrix} \quad (31)$$

$$\bar{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad (32)$$

and the gains $\bar{k}_1$, $\bar{k}_2$, $\bar{k}_3$ are yet to be chosen. We next state and prove the following result.

Theorem 2: Consider a homogenous platoon of vehicles with dynamics modeled by system (8), (9) under the control laws (28). There exists $\bar{K}$ such that each individual vehicular system is stable for any $D > 0$, and the platoon is $\mathscr{L}_p$, $p \in [1, \infty]$, string stable for any $D < h$.

Proof: See Appendix C. ■

Proposition 1: Condition $D < h$ is also necessary for the system to be simultaneously stable and string stable.

Proof: See Appendix D. ■

Remark 5: Although stability under the predictor-based ACC law with integral action is guaranteed for any delay value, string stability requires that the delay value is restricted to be smaller than the desired time-headway, which constitutes a considerable improvement compared to the string stability condition that the delay is smaller than half the time-headway imposed by other ACC designs (similar to the nominal, delay-free ACC law that we employ) without delay compensation, such as, for example, [52], [55]. The requirement of a constant time-headway policy that the steady-state spacing is greater than $Dv_{\rm d}$, where $v_{\rm d}$ is a desired speed dictated by the leader, is a physical limitation since during the $D-$second "dead-time" interval of the actuator the vehicle is not able to respond to large disturbances emanating from the preceding vehicle, e.g., rapid changes of its speed.† Thus, such a restriction is necessary in order for a vehicle to be able to attenuate disturbances imposed by its preceding vehicle and track it. Moreover, this limitation is in accordance to the result in [17] dealing with the disturbance attenuation limitations of systems with input delays under any time-invariant feedback controller.

It may be desirable, see, e.g., [6], that relation $\left|\bar{G}(j\omega)\right| < 1$, where $\bar{G}(s) = \frac{V_i(s)}{V_{i-1}(s)}$, $i = 1, \ldots, N$, holds for all $\omega > 0$, that is, that this inequality is strict. The following proposition shows that this holds true for the same $\bar{K}$ as in Theorem 2.

Proposition 2: Under the conditions of Theorem 2 it holds that $\left|\bar{G}(j\omega)\right| < 1$, for all $\omega > 0$, where $\bar{G}(s) = \frac{V_i(s)}{V_{i-1}(s)}$, $i = 1, \ldots, N$.

Proof: See Appendix E. ■

VI. String Stability Robustness of Predictor-Based Feedback to Delay Mismatch

We consider in this section the case where the delay D is not exactly known to the user, but there is an additive uncertainty to the nominal delay value employed in the predictor-based control law (28). Define $\Delta D = D_{\rm r} - D$, where $D_{\rm r}$ is the real value of the delay and D is an estimated value, available to the designer, of the actual delay $D_{\rm r}$. Thus, the new model for the vehicle dynamics is given by (8) together with

$$\dot{v}_i(t) = u_i\left(t - D_{\rm r}\right). \quad (33)$$

Theorem 3: Consider a homogenous platoon of vehicles with dynamics modeled by (8), (33) under the control laws (28). There exist $\bar{K}$ and a positive constant ϵ such that for all $|\Delta D| = |D_{\rm r} - D| < \epsilon$ each individual vehicular system is stable for any $D > 0$ and $D_{\rm r} \geq 0$. If, in addition, $D < h$, then the platoon is string stable in the $\mathscr{L}_2$ sense.

Proof: See Appendix F. ■

VII. Performance Evaluation of the Predictor-Based ACC With Integral Action

We consider the same scenario as in Section IV. In Fig. 8 we show the response of the four vehicles equipped with the developed predictor-based ACC law with integral action (28) with parameters $\bar{k}_1 = 14$, $\bar{k}_2 = 102$, $\bar{k}_3 = -20$, which satisfy the requirements of Theorem 2 (see Appendix C noting that $\bar{k}_1$, $\bar{k}_2$, and $\bar{k}_3$ are chosen according to (C.2)–(C.4) with $T_1 = 0.5$, $T_2 = 0.125$, and $T_3 = 0.1$, which satisfy (C.8), (C.9)). Since relation $D - h + T_2 + T_3 \leq 0$ should be satisfied (see (C.8) in Appendix C) one can observe that there is a limitation on the size of both T_2 and T_3, and thus, the choice $T_2 = 0.125$, $T_3 = 0.1$ is a reasonable choice within these

†Consider, e.g., the case in which the preceding vehicle comes to a complete stop instantaneously from a speed $v_{\rm d}$, then, due to the $D-$second "dead-time" interval of the actuator, the spacing between the two vehicles remains positive only if $D < h$ since the spacing satisfies $s(t) = (h - t)v_{\rm d}$, for all $t \leq D$.

bounds. Furthermore, since relation $D-h+T_1+T_3 \geq 0$ should be also satisfied (see (C.9) in Appendix C), it then follows that, approximately, $T_1 \geq 0.14$. Thus, we choose $T_1 = 0.5$ in order for this condition to be robustly satisfied and in order to achieve a quite fast response within this bound, as the speed of the response is determined by the dominant pole $-\frac{1}{T_1}$, thus compromising string stability robustness and speed response. One could choose a lower value for the constant T_1 for achieving a faster response, yet at the expense of making the controller less robust as far as string stability is concerned.

In general, according to the restrictions on T_1, T_2, T_3 (see (C.8), (C.9) in Appendix C) there is a trade-off between string stability robustness and speed response. To see this note that condition (C.8) is robustly satisfied for small T_2 and T_3. Yet, from (C.9) one can observe that the smaller the value for T_3 the larger the allowable value for T_1, and thus, the slower the speed of the response. On the contrary, larger values for T_3 allow smaller values for T_1, and thus, faster response, yet at the expense of bound (C.8) being satisfied more tightly.

One can observe from Fig. 8 that the four typical requirements of an ACC law, see, e.g., [12], namely a) stability of each individual vehicular system, b) zero steady-state spacing error, c) fulfillment of condition $\sup_{\omega\in\mathbb{R}} \left|\bar{G}(j\omega)\right| \leq 1$, and d) non-negative impulse response are satisfied.

We evaluate further and compare to the control law (26) the performance of the developed ACC design with integral action considering the following four physical requirements a) tracking error, b) safety, c) fuel consumption, and d) comfort. We consider a platoon of six vehicles and employ the following performance indices that quantify each of the four requirements

$$J_{\text{fuel}}=\sum_{i=1}^{6}\int_0^T J_i\left(v_i(t), a_i(t)\right) dt \tag{34}$$

$$J_i=\begin{cases}\beta_1+\beta_2 R_{T_i}\left(v_i(t), a_i(t)\right) v_i(t) & \\ +\beta_3 v_i(t) a_i(t)^2, & \text{if } R_{T_i}>0 \\ \beta_1, & \text{if } R_{T_i}\leq 0\end{cases} \tag{35}$$

$$R_{T_i}=\beta_4+\beta_5 v_i(t)^2+\beta_6 a_i(t) \tag{36}$$

$$J_{\text{comfort},1}=\sum_{i=1}^{6}\int_0^T \dot{a}_i(t)^2 dt \tag{37}$$

$$J_{\text{comfort},2}=\max_i \sup_{0\leq t\leq T} |\dot{a}_i(t)| \tag{38}$$

$$J_{\text{comfort},3}=\max_i \sup_{0\leq t\leq T} |a_i(t)| \tag{39}$$

$$J_{\text{safety}}=\sum_{i=1}^{6}\int_0^T \bar{J}_i\left(s_i(t), v_i(t), v_{i-1}(t)\right) dt \tag{40}$$

$$\bar{J}_i=\begin{cases} e^{\frac{1}{s_i(t)}}\left(v_{i-1}(t)-v_i(t)\right)^2, & \text{if } v_{i-1}(t)\leq v_i(t) \\ 0, & \text{otherwise}\end{cases} \tag{41}$$

$$J_{\text{tracking},1}=\sum_{i=1}^{6}\int_0^T \delta_i(t)^2 dt \tag{42}$$

$$J_{\text{tracking},2}=\sum_{i=1}^{6}\int_0^T \left(v_i(t)-v_{i-1}(t)\right)^2 dt, \tag{43}$$

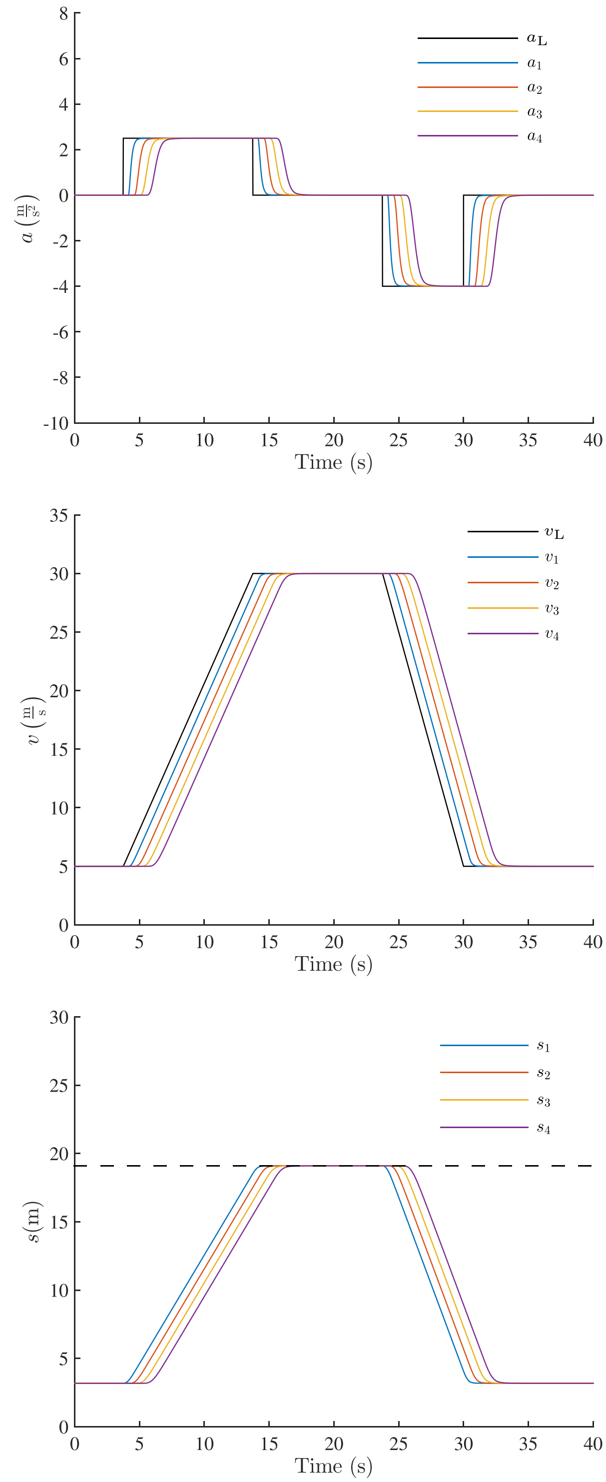

Fig. 8. Acceleration (top), speed (middle), and spacing (bottom) of four vehicles following a leader that performs the acceleration maneuver shown in Fig. 3, under the delay-compensating ACC strategy with integral action (28). The reference spacing $\frac{2}{\pi}\times 30 \approx 19.1$ m is depicted with dashed line.

which are used in the literature, see, e.g., [1], [29], [48]. We choose $T = 40$ s, whereas the parameters of (34) are shown in Table I. The percentage improvements of each cost when the proposed ACC design (28) is employed in comparison to the case where the ACC law (26) is utilized

TABLE I
PARAMETERS OF THE FUEL CONSUMPTION COST (34)

Parameter	Value
β_1	0.666
β_2	0.0717
β_3	0.0578
β_4	0.527
β_5	0.000948
β_6	1.68

TABLE II
PERFORMANCE INDICES (34), (37)–(40), (42), AND (43)

Performance index	Percentage improvement with (28) in comparison to (26)
J_{fuel}	28
$J_{\mathrm{comfort},1}$	90
$J_{\mathrm{comfort},2}$	20
$J_{\mathrm{comfort},3}$	66
J_{safety}	53
$J_{\mathrm{tracking},1}$	83
$J_{\mathrm{tracking},2}$	51

are shown in Table II. It is evident that the predictor-based ACC law with integral action achieves better performance in all metrics. Note that the performance improvement with the delay-compensating ACC law (28) compared to the control law (26) would be larger when one considers a larger number of vehicles in the platoon due to the lack of string stability in the case of the uncompensated control law (26).

Note that in the simulation investigation we consider initial conditions such that the system is at equilibrium. The stability and performance guarantees of predictor-based control laws due to non-zero initial conditions are well-studied, see, e.g., [5], [18]. In the present case, we expect that although the proposed ACC design guarantees all four typical requirements of an ACC law, there would be a trade-off between the transient performance in response to leader's maneuver and the transient performance in response to initial conditions that are not at equilibrium. For example, in the latter case, the control input, i.e., the desired acceleration, may exhibit a large pick when it "kicks in" at $t = D$, which is a result of the high gains employed by the controller. Note that such high gains are necessary to guarantee string stability. To see this, note that the conditions for string stability (see (C.8), (C.9) in Appendix C) impose an upper bound on the allowable values for T_2, T_3, and thus, a lower bound on the gains $\bar{k}_1$, $\bar{k}_2$, and $\bar{k}_3$ (see relations (C.2)–(C.4) in Appendix C).

In an actual implementation, this may imply that a warm-up period may be necessary before the proposed ACC law could start operating. During this period, one may, for instance, employ a switching logic, in which, the gains of the proposed ACC law are switched to lower values (that may guarantee only vehicle stability, which is ensured for any positive T_1, T_2, T_3), until the effect of non-zero initial conditions has faded away.

VIII. CONCLUSIONS AND DISCUSSION

We presented a predictor-based ACC design methodology for compensation of long input delays in vehicular systems. We showed that the developed ACC algorithm guarantees that four of the most common performance requirements of ACC designs are satisfied. The performance of the proposed ACC strategy is verified in simulation considering various quantitative performance measures. A potential future research path is to consider the case of heterogenous platoons.

One might raise the question of combination of the predictor principle with other, more complicated nominal ACC strategies than the nominal control law employed in this paper, which may incorporate additional information about the state of the preceding vehicle, such as, for example, its speed. Such an extension may be possible employing CACC systems, which have the capability of providing a vehicle with measurements of the control action of other vehicles through V2V communication, such as, for example, their acceleration. The reason that a CACC system is needed is the following. In order for the controller of a vehicle to be able to predict the future values of certain states of the preceding vehicle, such as, for example, its speed, measurements of the control input of the preceding vehicle, for example, its acceleration, are needed.

Since nonlinear predictor-based feedback control laws have already been developed, see, e.g., [5], [17], [23], a possible future research direction is to consider nonlinear models of vehicular systems and employ nonlinear nominal ACC laws.

APPENDIX A
REVIEW OF THE BASIC PRINCIPLES OF PREDICTOR FEEDBACK

We review here the basic ideas of predictor feedback as these are presented from [5, Ch. 2.1.1].

Consider a linear system with input delay

$$\dot{x}(t) = Ax(t) + Bu(t - D), \tag{A.1}$$

where $x \in \mathbb{R}^n$, (A, B) is a controllable pair, and $D > 0$ is arbitrarily long. When $D = 0$, a control law for system (A.1) is given as

$$u(t) = Kx(t), \tag{A.2}$$

where the gain K is chosen such that the matrix $A + BK$ has the desired eigenvalues. The predictor feedback design is based on the nominal delay-free design (A.2).

The main idea is to replace the state x by its prediction over a D-time-unit horizon, namely the signal

$$p(t) = x(t + D), \tag{A.3}$$

so that $u(t - D) = Kx(t)$. The key challenge is how to derive an implementable form of the signal p, i.e., a form that does not incorporate the future values of the state x, as they are not available for feedback. Having determined the predictor p of the state x, the control law for the system with delay is given by

$$u(t) = Kp(t). \tag{A.4}$$

This control law completely compensates the input delay since for all $t \geq D$, $u(t - D) = Kp(t - D) = Kx(t)$, and hence, the closed-loop system behaves as if there were no delay at all (after an initial transient period of D time-units).

An implementable form of the predictor signal is derived employing model (A.1) and the variation of constants formula. One can see this as follows. Performing the change of variables $t = \theta + D$, for all $t - D \leq \theta \leq t$, in (A.1) and using the fact that $\frac{d\theta}{dt} = 1$, we get for all $t - D \leq \theta \leq t$

$$\frac{dx(\theta + D)}{d\theta} = Ax(\theta + D) + Bu(\theta). \tag{A.5}$$

Defining the new signal

$$p(\theta) = x(\theta + D), \text{for all } t - D \leq \theta \leq t, \tag{A.6}$$

and solving the resulting ODE in θ for p with initial condition $p(t - D) = x(t)$ we arrive at

$$p(\theta) = e^{A(\theta - t + D)}x(t) + \int_{t-D}^{\theta} e^{A(\theta - s)}Bu(s)ds, \tag{A.7}$$

for all $t - D \leq \theta \leq t$, and hence,

$$p(t) = e^{AD}x(t) + \int_{t-D}^{t} e^{A(t-\theta)}Bu(\theta)d\theta. \tag{A.8}$$

Representation (A.8) for the predictor signal is directly implementable since it is given in terms of the measured state $x(t)$ and the history of $u(\theta)$, for all $t - D \leq \theta \leq t$. Note also that $p(\theta)$ in (A.7) should be viewed as the output of an operator, parametrized by t, acting on $u(s)$, $t - D \leq s \leq \theta$, in the same way that the solution $x(t)$ to a linear ODE (i.e., $x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-s)}Bu(s)ds$) can be viewed as the output of an operator, parametrized by t_0, acting on $u(s)$, $t_0 \leq s \leq t$.

The final predictor feedback law is given by combining (A.4) and (A.8) as

$$u(t) = K\left(e^{AD}x(t) + \int_{t-D}^{t} e^{A(t-\theta)}Bu(\theta)d\theta\right). \tag{A.9}$$

One can view the feedback law (A.9) as implicit, since u appears on both (left and right) sides of the equation. However, one should observe that the input memory $u(\theta)$, $\theta \in [t - D, t]$ is a part of the state of the overall infinite-dimensional system, so the control law is in fact given by an explicit full-state feedback formula.

Appendix B
Proof of Theorem 1

We start by deriving the transfer function

$$G(s) = \frac{V_i(s)}{V_{i-1}(s)}, i = 1, \ldots, N, \tag{B.1}$$

viewing the preceding vehicle's speed as input and the current vehicle's speed as output, see, e.g., [6], [10], [24]. In view of Remark 1, for studying stability and string stability under the predictor-based control law, it is sufficient to study the properties of G.

Taking the Laplace transform of the control law (12) we get

$$U_i(s) = Ke^{\Gamma D}X_i(s) + M(s)U_i(s) \tag{B.2}$$
$$M(s) = K\left(sI_{2\times 2} - \Gamma\right)^{-1}\left(I_{2\times 2} - e^{\Gamma D}e^{-sD}\right)B, \tag{B.3}$$

where we used the fact that

$$\begin{aligned}\mathcal{L}&\left\{K\int_{t-D}^{t} e^{\Gamma(t-\theta)}Bu_i(\theta)d\theta\right\}\\ &= \mathcal{L}\left\{K\int_{0}^{D} e^{\Gamma(D-y)}Bu_i(t+y-D)dy\right\}\\ &= K\left(sI_{2\times 2} - \Gamma\right)^{-1}\left(I_{2\times 2} - e^{(\Gamma - sI_{2\times 2})D}\right)BU_i(s).\end{aligned} \tag{B.4}$$

Using the i-th vehicle's model (8), (9) we have

$$X_i(s) = \left(sI_{2\times 2} - \Gamma\right)^{-1}\left(Be^{-sD}U_i(s) + B_v V_{i-1}(s)\right), \tag{B.5}$$

where

$$B_v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{B.6}$$

Substituting (B.5) into (B.2) we get that

$$U_i(s) = \frac{K\left(sI_{2\times 2} - \Gamma\right)^{-1}e^{\Gamma D}B_v}{1 - K\left(sI_{2\times 2} - \Gamma\right)^{-1}B}V_{i-1}(s), \tag{B.7}$$

and thus, from (B.5) we arrive at

$$X_i(s) = R(s)V_{i-1}(s), \tag{B.8}$$

where

$$\begin{aligned}R(s) = &\frac{\left(sI_{2\times 2} - \Gamma\right)^{-1}}{1 - K\left(sI_{2\times 2} - \Gamma\right)^{-1}B}\Big(B_v + Be^{-sD}\\ &\times K\left(sI_{2\times 2} - \Gamma\right)^{-1}e^{\Gamma D}B_v - K\left(sI_{2\times 2} - \Gamma\right)^{-1}\\ &\times BB_v\Big).\end{aligned} \tag{B.9}$$

Note that due to (B.9), the spectrum of the closed-loop system is finite [14], [28]. Using the facts that

$$e^{\Gamma D} = \begin{bmatrix} 1 & -D \\ 0 & 1 \end{bmatrix} \tag{B.10}$$
$$\left(sI_{2\times 2} - \Gamma\right)^{-1} = \frac{1}{s^2}\begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}, \tag{B.11}$$

and multiplying (B.8) from the left with $\begin{bmatrix} 0 & 1 \end{bmatrix}$ we obtain

$$G(s) = \frac{\frac{\alpha}{h}e^{-Ds}}{s^2 + \alpha s + \frac{\alpha}{h}}. \tag{B.12}$$

Stability: From the denominator of G in (B.12) it follows that for any positive α and h the transfer function G is asymptotically stable (see also [5], [18], [23] for detailed studies on the stability properties of predictor-based feedbacks).

String stability in the $\mathscr{L}_2$ sense: String stability in the $\mathscr{L}_2$ sense is guaranteed when $\sup_{\omega\in\mathbb{R}} |G(j\omega)| \leq 1$, see, e.g., [6]. Using (B.12) we obtain the condition

$$\frac{\frac{\alpha^2}{h^2}}{\left(\frac{\alpha}{h} - \omega^2\right)^2 + \alpha^2\omega^2} \leq 1, \text{for all } \omega \in \mathbb{R}, \tag{B.13}$$

which is satisfied when the following holds

$$\omega^4 + \omega^2\alpha\left(\alpha - \frac{2}{h}\right) \geq 0, \text{for all } \omega \in \mathbb{R}. \tag{B.14}$$

Relation (B.14) holds when $\alpha \geq \frac{2}{h}$.

String stability in the $\mathscr{L}_p$, $p \in [1, \infty]$, sense: The impulse response of the transfer function G defined in (B.12) is given by

$$g(t) = \begin{cases} 0, & 0 \le t \le D \\ f(t-D), & t \ge D \end{cases}, \tag{B.15}$$

where f is the impulse response of the delay-free system under the nominal control design, i.e.,

$$f(t) = \mathcal{L}^{-1}\left\{ \frac{\frac{\alpha}{h}}{s^2 + \alpha s + \frac{\alpha}{h}} \right\}.$$

Choosing $\alpha > \frac{4}{h}$ the characteristic polynomial $s^2 + \alpha s + \frac{\alpha}{h}$ has two distinct real roots, say, p_1 and p_2, and hence,

$$g(t) = \begin{cases} 0, & 0 \le t \le D \\ \frac{\alpha}{h(p_1 - p_2)}\left(e^{p_1(t-D)} - e^{p_2(t-D)}\right), & t \ge D \end{cases} \tag{B.16}$$

$$\ge 0, \text{ for all } t \ge 0. \tag{B.17}$$

Since $|G(0)| = 1$, we get from (B.17) that the system is string stable in the $\mathscr{L}_p$, $p \in [1, \infty]$, sense, see, e.g., [6]. For $\alpha = \frac{4}{h}$ the impulse response (B.16) becomes

$$g(t) = \begin{cases} 0, & 0 \le t \le D \\ \frac{\alpha}{h}(t-D)e^{-\frac{2}{h}(t-D)}, & t \ge D \end{cases} \tag{B.18}$$

$$\ge 0, \text{ for all } t \ge 0. \tag{B.19}$$

Thus, employing the same arguments with the case $\alpha > \frac{4}{h}$, one can conclude that the system is string stable also for $\alpha = \frac{4}{h}$.

Appendix C
Proof of Theorem 2

With similar computations as in the proof of Theorem 1, we obtain that the transfer function $\bar{G}(s) = \frac{V_i(s)}{V_{i-1}(s)}$, $i = 1, \ldots, N$, is given by

$$\bar{G}(s) = \frac{\left(\left(D + \frac{h\bar{k}_1}{\bar{k}_2}\right)s + 1\right)e^{-sD}}{\frac{h}{\bar{k}_2}s^3 - \frac{h\bar{k}_3}{\bar{k}_2}s^2 + \frac{h(\bar{k}_1 + \bar{k}_2)}{\bar{k}_2}s + 1}. \tag{C.1}$$

Stability: From (C.1) it follows that there exists choice of $\bar{K}$ that renders the transfer function $\bar{G}$ asymptotically stable. One can see this by matching the denominator of $\bar{G}$ in (C.1) with any desired third-order polynomial of the form $(T_1 s + 1)(T_2 s + 1)(T_3 s + 1)$, where $T_1 > T_2 > T_3 > 0$, that is, the parameters $\bar{k}_1, \bar{k}_2, \bar{k}_3$ can be chosen as

$$\bar{k}_1 = \frac{T_1 + T_2 + T_3 - h}{T_1 T_2 T_3} \tag{C.2}$$

$$\bar{k}_2 = \frac{h}{T_1 T_2 T_3} \tag{C.3}$$

$$\bar{k}_3 = -\frac{T_1 T_2 + T_1 T_3 + T_2 T_3}{T_1 T_2 T_3}. \tag{C.4}$$

String stability in the $\mathscr{L}_p$, $p \in [1, \infty]$, sense: The impulse response of the transfer function $\bar{G}$ defined in (C.1) is given by

$$\bar{g}(t) = \begin{cases} 0, & 0 \le t \le D \\ \bar{f}(t-D), & t \ge D \end{cases}, \tag{C.5}$$

where $\bar{f}$ is the impulse response of the delay-free system under the nominal control design, i.e.,

$$\bar{f}(t) = \mathcal{L}^{-1}\left\{ \frac{\left(D + \frac{h\bar{k}_1}{\bar{k}_2}\right)s + 1}{\frac{h}{\bar{k}_2}s^3 - \frac{h\bar{k}_3}{\bar{k}_2}s^2 + \frac{h(\bar{k}_1 + \bar{k}_2)}{\bar{k}_2}s + 1} \right\}.$$

Selecting the parameters $\bar{k}_1, \bar{k}_2, \bar{k}_3$ such that $T_1 > T_2 > T_3 > 0$, we conclude from [26] (Theorem 5) that $\bar{f}(t) \ge 0$, for all $t \ge 0$, when the following hold (see also [3] for similar conditions)

$$D + \frac{h\bar{k}_1}{\bar{k}_2} \le T_1 \tag{C.6}$$

$$D + \frac{h\bar{k}_1}{\bar{k}_2} \ge T_2. \tag{C.7}$$

Using (C.2)–(C.4), conditions (C.6), (C.7) are rewritten as

$$D - h + T_2 + T_3 \le 0 \tag{C.8}$$

$$D - h + T_1 + T_3 \ge 0. \tag{C.9}$$

Conditions (C.8), (C.9) can be satisfied by an appropriate choice of $T_1 > T_2 > T_3 > 0$ when $D < h$. Since also $\left|\bar{G}(0)\right| = 1$, we conclude that the system is string stable in the $\mathscr{L}_p$, $p \in [1, \infty]$, sense, see, e.g., [6].

Appendix D
Proof of Proposition 1

We prove this fact by showing that the $\mathscr{L}_2$ string stability condition that $\sup_{\omega \in \mathbb{R}} \left|\bar{G}(j\omega)\right| \le 1$, $\omega \ne 0$, may be violated when $D \ge h$. We start by observing from (C.1) that $\bar{G}$ is asymptotically stable when the following hold

$$\bar{k}_2 > 0 \tag{D.1}$$

$$\bar{k}_3 < 0 \tag{D.2}$$

$$\frac{\bar{k}_2}{h} + \bar{k}_3\left(\bar{k}_1 + \bar{k}_2\right) < 0. \tag{D.3}$$

Condition $\sup_{\omega \in \mathbb{R}} \left|\bar{G}(j\omega)\right| \le 1$, for all $\omega \ge 0$, is equivalent to

$$\omega^4 + \alpha_1 \omega^2 + \alpha_2 \ge 0, \text{ for all } \omega > 0, \tag{D.4}$$

where

$$\alpha_1 = \bar{k}_3^2 - 2\left(\bar{k}_1 + \bar{k}_2\right) \tag{D.5}$$

$$\alpha_2 = \bar{k}_2^2\left(1 - \frac{D^2}{h^2}\right) + 2\bar{k}_2\left(\frac{\bar{k}_3}{h} + \bar{k}_1\left(1 - \frac{D}{h}\right)\right). \tag{D.6}$$

Assume first that $D = h$. Then, from (D.1), (D.2) it follows that the constant term of the second-order polynomial in ω^2 in the left-hand side of inequality (D.4) must be negative, which implies that there exists a sufficiently small μ such that, for all $0 < \omega < \mu$, relation (D.4) cannot hold.[‡] Similarly, in the case where $D > h$, we rewrite the constant term of the second-order polynomial in ω^2 in the left-hand side of inequality (D.4) as

$$\bar{k}_2\left(\left(1 - \frac{D}{h}\right)\left(2\bar{k}_1 + \bar{k}_2\left(1 + \frac{D}{h}\right)\right) + 2\frac{\bar{k}_3}{h}\right), \tag{D.7}$$

[‡]Take, for instance, $\mu = \sqrt{\frac{-\alpha_1 + \sqrt{\alpha_1^2 - 4\alpha_2}}{2}}$ when $\alpha_1 > 0$ and $\mu = (-\alpha_2)^{\frac{1}{4}}$ when $\alpha_1 \le 0$.

which is negative (since from (D.3) it follows that $\bar{k}_1+\bar{k}_2 > 0$, which also implies that $2\bar{k}_1 + \bar{k}_2\left(1 + \frac{D}{h}\right)$ is positive as well), and thus, there exists a sufficiently small μ such that, for all $0 < \omega < \mu$, relation (D.4) cannot hold for this case either.

Appendix E
Proof of Proposition 2

It is verified that relation $\left|\bar{G}(j\omega)\right| < 1$ holds for all $\omega > 0$ when the parameters of the controller $\bar{k}_1, \bar{k}_2, \bar{k}_3$ are chosen according to (C.2)–(C.4) and when conditions (C.8), (C.9) are satisfied, since in this case both α_1 and α_2 are positive. This can be shown by utilizing (C.2)–(C.4) to re-write α_1 in (D.5) as $\alpha_1 = \frac{1}{T_3^2} + \frac{1}{T_2^2} + \frac{1}{T_1^2}$ and by performing some algebraic manipulations to conclude from (D.6) that α_2 is positive when the second-order polynomial in $h - D$ defined as $(h-D)^2 - 2(T_1+T_2+T_3)(h-D) + 2(T_1T_2+T_1T_3+T_2T_3)$ is negative for all $T_2+T_3 \le h-D \le T_1+T_3$, which in turn holds true since its roots are $T_1+T_2+T_3 \pm \sqrt{T_1^2+T_2^2+T_3^2}$.

Appendix F
Proof of Theorem 3

Consider a $\bar{K}$ given by (C.2)–(C.4), with any T_1, T_2, T_3 such that $T_1 > T_2 > T_3 > 0$. From [16] and [22] it follows that there exists a positive constant ϵ_1 such that for any $|\Delta D| < \epsilon_1$ each individual vehicular system is stable.

We turn our attention next to proving $\mathscr{L}_2$ string stability. Assume that $D < h$ and consider again a $\bar{K}$ given by (C.2)–(C.4), where $T_1 > T_2 > T_3 > 0$ satisfy (C.8), (C.9). Repeating the computations of the proof of Theorems 1, 2 and using (C.2)–(C.4), we derive the new transfer function $G_1(s) = \frac{V_i(s)}{V_{i-1}(s)}$ as

$$G_1(s)=\frac{(T_1+T_2+T_3+D-h)\,s+1}{\pi(s)+w(s)}e^{-sD_{\mathrm{r}}} \tag{F.1}$$

$$\pi(s)=T_1T_2T_3s^3+(T_1T_2+T_1T_3+T_2T_3)\,s^2 + (T_1+T_2+T_3)\,s+1 \tag{F.2}$$

$$w(s)=\left(\rho_1 s^2+(T_1+T_2+T_3+D)\,s+1\right) \times\left(e^{-sD_{\mathrm{r}}}-e^{-sD}\right) \tag{F.3}$$

$$\rho_1=\left(D\,(T_1+T_2+T_3)+\frac{D^2}{2} + T_1T_2+T_1T_3+T_2T_3\right). \tag{F.4}$$

Thus,

$$|G_1(j\omega)|^2=\frac{1+(T_1+T_2+T_3+D-h)^2\,\omega^2}{f_1^2(\omega)+f_2^2(\omega)} \tag{F.5}$$

$$\begin{aligned} f_1(\omega)=&1-\omega^2\,(T_1T_2+T_1T_3+T_2T_3)\\ &-\left(1-\rho_1\omega^2\right)(\cos(\omega D)-\cos(\omega D_{\mathrm{r}}))\\ &-\omega\,(T_1+T_2+T_3+D)\\ &\times\,(\sin(\omega D)-\sin(\omega D_{\mathrm{r}})) \end{aligned} \tag{F.6}$$

$$\begin{aligned} f_2(\omega)=&-T_1T_2T_3\omega^3+\omega\,(T_1+T_2+T_3)\\ &-\omega\,(T_1+T_2+T_3+D)\,(\cos(\omega D)-\cos(\omega D_{\mathrm{r}}))\\ &+\left(1-\rho_1\omega^2\right)(\sin(\omega D)-\sin(\omega D_{\mathrm{r}}))\,. \end{aligned} \tag{F.7}$$

String stability is guaranteed when $|G_1(j\omega)|^2 \le 1$, for all $\omega \ge 0$. The condition is satisfied for $\omega = 0$ since $|G_1(0)| = 1$. Therefore, the condition for string stability becomes

$$f_1^2(\omega)+f_2^2(\omega)-1-(T_1+T_2+T_3+D-h)^2\,\omega^2 \ge 0, \quad \text{for all } \omega > 0. \tag{F.8}$$

We consider next two distinct cases.

Case $\Delta D < 0$: For a given ω, from the mean-value theorem we conclude that there exist $\xi(\omega)$ and $\zeta(\omega)$ such that

$$\cos(\omega D)-\cos(\omega D_{\mathrm{r}}) = \omega\Delta D\sin(\omega\xi(\omega)), \quad \xi\in(D_{\mathrm{r}}, D) \tag{F.9}$$

$$\sin(\omega D)-\sin(\omega D_{\mathrm{r}}) = -\omega\Delta D\cos(\omega\zeta(\omega)), \quad \zeta\in(D_{\mathrm{r}}, D)\,. \tag{F.10}$$

Therefore, (F.6), (F.7) can be written as

$$\begin{aligned} f_1(\omega) =& 1-\omega^2\,(T_1T_2+T_1T_3+T_2T_3)\\ &-\left(1-\rho_1\omega^2\right)\omega\Delta D\sin(\omega\xi(\omega)))\\ &+\omega^2\,(T_1+T_2+T_3+D)\\ &\times\,\Delta D\cos(\omega\zeta(\omega)) \end{aligned} \tag{F.11}$$

$$\begin{aligned} f_2(\omega) =& -T_1T_2T_3\omega^3+\omega\,(T_1+T_2+T_3)\\ &-\omega^2\,(T_1+T_2+T_3+D)\,\Delta D\sin(\omega\xi(\omega)))\\ &-\left(1-\rho_1\omega^2\right)\omega\Delta D\cos(\omega\zeta(\omega))\,. \end{aligned} \tag{F.12}$$

Using (F.5)–(F.12) and performing some algebraic manipulations one can conclude that string stability is guaranteed when

$$\begin{aligned} &q_5(\Delta D,\omega)\,\omega^5+q_4(\Delta D,\omega)\,\omega^4\\ &\quad+q_3(\Delta D,\omega)\,\omega^3+q_2(\Delta D,\omega)\,\omega^2\\ &\quad+q_1(\Delta D,\omega)\,\omega+q_0(\Delta D,\omega)\ge 0, \forall\omega>0, \end{aligned} \tag{F.13}$$

where

$$\begin{aligned} q_5(\Delta D,\omega)=&(T_1T_2T_3-\Delta D\rho_1\cos(\omega\zeta(\omega)))^2\\ &+\Delta D^2\rho_1^2\sin^2(\omega\xi(\omega)) \end{aligned} \tag{F.14}$$

$$\begin{aligned} q_4(\Delta D,\omega)=&2T_1T_2T_3\,(T_1+T_2+T_3+D)\,\Delta D\\ &\times\,\sin(\omega\xi(\omega))-2\Delta D\rho_1\,(T_1T_2+T_1T_3\\ &+T_2T_3)\sin(\omega\xi(\omega)) \end{aligned} \tag{F.15}$$

$$\begin{aligned} q_3(\Delta D,\omega)=&(T_1T_2+T_1T_3+T_2T_3-(T_1+T_2+T_3+D)\\ &\times\,\Delta D\cos(\omega\zeta(\omega)))^2\\ &-2\,(T_1T_2T_3-\Delta D\rho_1\cos(\omega\zeta(\omega)))\\ &\times\,(T_1+T_2+T_3-\Delta D\cos(\omega\zeta(\omega)))\\ &+\,(T_1+T_2+T_3+D)^2\,\Delta D^2\sin^2(\omega\xi(\omega))\\ &-2\Delta D^2\rho_1\sin^2(\omega\xi(\omega)) \end{aligned} \tag{F.16}$$

$$\begin{aligned} q_2(\Delta D,\omega)=&2\Delta D\rho_1\sin(\omega\xi(\omega))+2\Delta D\sin(\omega\xi(\omega))\\ &\times\,(T_1T_2+T_1T_3+T_2T_3)-2\Delta D\sin(\omega\xi(\omega))\\ &\times\,(T_1+T_2+T_3)\,(T_1+T_2+T_3+D) \end{aligned} \tag{F.17}$$

$$q_1(\Delta D,\omega)=(T_1+T_2+T_3-\Delta D\cos(\omega\zeta(\omega)))^2 -2(T_1T_2+T_1T_3+T_2T_3)+\Delta D^2 \times \sin^2(\omega\xi(\omega))-(T_1+T_2+T_3+D-h)^2 +2\Delta D\cos(\omega\zeta(\omega))(T_1+T_2+T_3+D) \quad \text{(F.18)}$$

$$q_0(\Delta D,\omega)=-2\Delta D\sin(\omega\xi(\omega)). \quad \text{(F.19)}$$

Next, we lower-bound functions q_0, q_2, q_4. Using the facts that $|\sin(x)|\le|x|$, for all $x\in\mathbb{R}$, that $\omega>0$, that $\Delta D<0$, and that $0\le\xi<D$ we get

$$q_0(\Delta D,\omega) \geq 2D\Delta D\omega. \quad \text{(F.20)}$$

Similarly, it follows that

$$q_2(\Delta D,\omega) \geq 2D\omega\Delta D\gamma_1 \quad \text{(F.21)}$$
$$q_4(\Delta D,\omega) \geq 2D\omega\Delta D\gamma_2 \quad \text{(F.22)}$$
$$\gamma_1 = \left|\frac{D^2}{2}-T_1^2-T_2^2-T_3^2\right| \quad \text{(F.23)}$$
$$\gamma_2 = |T_1T_2T_3(T_1+T_2+T_3+D) -\rho_1(T_1T_2+T_1T_3+T_2T_3)|, \quad \text{(F.24)}$$

where we also used (F.4). Therefore, relation (F.13) holds when

$$\bar{q}_3(\Delta D,\omega)\omega^4+\bar{q}_2(\Delta D,\omega)\omega^2+\bar{q}_1(\Delta D,\omega)\ge 0, \forall\omega>0, \quad \text{(F.25)}$$

where

$$\bar{q}_1(\Delta D,\omega) = q_1(\Delta D,\omega)+2D\Delta D \quad \text{(F.26)}$$
$$\bar{q}_2(\Delta D,\omega) = q_3(\Delta D,\omega)+2D\Delta D\gamma_1 \quad \text{(F.27)}$$
$$\bar{q}_3(\Delta D,\omega) = q_5(\Delta D,\omega)+2D\Delta D\gamma_2. \quad \text{(F.28)}$$

We next show that $\bar{q}_1$, $\bar{q}_2$, and $\bar{q}_3$ are positive for sufficiently small ΔD. Combining (F.26)–(F.28) with (F.14), (F.16), and (F.18), we obtain that for all $\omega\in\mathbb{R}$ the following hold

$$\bar{q}_1(\Delta D,\omega) \geq q_1^*(\Delta D) \quad \text{(F.29)}$$
$$\bar{q}_2(\Delta D,\omega) \geq q_2^*(\Delta D) \quad \text{(F.30)}$$
$$\bar{q}_3(\Delta D,\omega) \geq q_3^*(\Delta D), \quad \text{(F.31)}$$

where

$$q_1^*(\Delta D) = T_1^2+T_2^2+T_3^2 -(D-h+T_1+T_2+T_3)^2 +4\Delta D(D+T_1+T_2+T_3) \quad \text{(F.32)}$$
$$q_2^*(\Delta D) = T_1^2T_2^2+T_1^2T_3^2+T_2^2T_3^2 -4\Delta D^2\rho_1+2\Delta D(D\gamma_1+T_1T_2T_3 +\rho_1(T_1+T_2+T_3)+(D+T_1+T_2+T_3) \times(T_1T_2+T_1T_3+T_2T_3)) \quad \text{(F.33)}$$
$$q_3^*(\Delta D) = T_1^2T_2^2T_3^2+2\Delta D(T_1T_2T_3\rho_1+D\gamma_2). \quad \text{(F.34)}$$

Quantity $T_1^2+T_2^2+T_3^2-(D-h+T_1+T_2+T_3)^2$ may be viewed as a second-order polynomial in $h-D$, and thus, it can be written as

$$T_1^2+T_2^2+T_3^2-(D-h+T_1+T_2+T_3)^2 = -\left(h-D-(T_1+T_2+T_3)-\sqrt{T_1^2+T_2^2+T_3^2}\right) \times\left(h-D-(T_1+T_2+T_3)+\sqrt{T_1^2+T_2^2+T_3^2}\right). \quad \text{(F.35)}$$

Therefore, from (C.8), (C.9) it follows that

$$T_1^2+T_2^2+T_3^2-(D-h+T_1+T_2+T_3)^2 \ge\left(T_2+\sqrt{T_1^2+T_2^2+T_3^2}\right)\times\left(-T_1+\sqrt{T_1^2+T_2^2+T_3^2}\right), \quad \text{(F.36)}$$

and thus,

$$T_1^2+T_2^2+T_3^2-(D-h+T_1+T_2+T_3)^2>0. \quad \text{(F.37)}$$

Moreover, using (F.32)–(F.34), (F.37) one can conclude that q_1^*, q_2^*, and q_3^* are continuous functions of ΔD with $q_1^*(0)>0$, $q_2^*(0)>0$, and $q_3^*(0)>0$. Thus, for sufficiently small ΔD the functions q_1^*, q_2^*, and q_3^*, and consequently, via (F.29)–(F.31) the functions $\bar{q}_1$, $\bar{q}_2$, and $\bar{q}_3$, are positive. One can choose, for example, $0<-\Delta D<\epsilon_2$, with

$$\epsilon_2=\min\left\{\epsilon_1^*,\epsilon_2^*,\epsilon_3^*\right\}, \quad \text{(F.38)}$$

where

$$\epsilon_1^*=\frac{T_1^2+T_2^2+T_3^2-(D-h+T_1+T_2+T_3)^2}{4(D+T_1+T_2+T_3)} \quad \text{(F.39)}$$
$$\epsilon_2^*=\frac{1}{2}\left(-b_1+\sqrt{b_1^2+\frac{b_2}{\rho_1}}\right) \quad \text{(F.40)}$$
$$\epsilon_3^*=\frac{T_1^2T_2^2T_3^2}{2(T_1T_2T_3\rho_1+D\gamma_2)} \quad \text{(F.41)}$$
$$b_1=\frac{1}{2\rho_1}(D\gamma_1+T_1T_2T_3 +\rho_1(T_1+T_2+T_3)+(D+T_1+T_2+T_3) \times(T_1T_2+T_1T_3+T_2T_3)) \quad \text{(F.42)}$$
$$b_2=T_1^2T_2^2+T_1^2T_3^2+T_2^2T_3^2. \quad \text{(F.43)}$$

Case $\Delta D>0$: The proof in this case follows employing similar arguments to the case $\Delta D<0$. In particular, relations (F.9), (F.10) hold with $\xi\in(D,D_{\mathrm{r}})$ and $\zeta\in(D,D_{\mathrm{r}})$, respectively. Thus, relations (F.11)–(F.19) hold. Moreover, since $0<D<D_{\mathrm{r}}$ relations (F.20)–(F.22) hold with D being replaced by $-D_{\mathrm{r}}$. Thus, relation (F.25) holds where in (F.26)–(F.28) D is replaced by $-D_{\mathrm{r}}$. Furthermore, relations (F.29)–(F.31) hold as well where, using the fact that $D_{\mathrm{r}}=\Delta D+D$, relations (F.32)–(F.34) are modified to

$$q_1^*(\Delta D)=T_1^2+T_2^2+T_3^2 -(D-h+T_1+T_2+T_3)^2 -4\Delta D(D+T_1+T_2+T_3)-2\Delta D^2 \quad \text{(F.44)}$$
$$q_2^*(\Delta D)=T_1^2T_2^2+T_1^2T_3^2+T_2^2T_3^2 -2\Delta D^2(2\rho_1+\gamma_1)-2\Delta D(D\gamma_1+T_1T_2T_3 +\rho_1(T_1+T_2+T_3)+(D+T_1+T_2+T_3) \times(T_1T_2+T_1T_3+T_2T_3)) \quad \text{(F.45)}$$
$$q_3^*(\Delta D)=T_1^2T_2^2T_3^2-2\Delta D(T_1T_2T_3\rho_1+D\gamma_2) -2\Delta D^2\gamma_2. \quad \text{(F.46)}$$

With similar arguments as in the case $\Delta D<0$ we obtain that (F.25) holds when ΔD is chosen such that $0<\Delta D<\epsilon_3$, where

$$\epsilon_3=\min\left\{\delta_1^*,\delta_2^*,\delta_3^*\right\}, \quad \text{(F.47)}$$

with

$$\delta_1^*=\frac{1}{2}\left(-2r_2+\sqrt{4r_2^2+2r_1}\right) \tag{F.48}$$

$$\delta_2^*=\frac{1}{2}\left(-\frac{2b_1\rho_1}{r_4}+\sqrt{\frac{4b_1^2r_1^2}{r_4^2}+2\frac{r_3}{r_4}}\right) \tag{F.49}$$

$$\delta_3^*=\begin{cases}\frac{1}{2}\left(-\frac{r_6}{\gamma_2}+\sqrt{\frac{r_6^2}{\gamma_2^2}+2\frac{r_5}{\gamma_2}}\right), & \gamma_2\neq 0\\ \frac{T_1T_2T_3}{2\rho_1}, & \gamma_2=0\end{cases} \tag{F.50}$$

$$r_1=T_1^2+T_2^2+T_3^2-(D-h+T_1+T_2+T_3)^2 \tag{F.51}$$

$$r_2=D+T_1+T_2+T_3 \tag{F.52}$$

$$r_3=T_1^2T_2^2+T_1^2T_3^2+T_2^2T_3^2 \tag{F.53}$$

$$r_4=2\rho_1+\gamma_1 \tag{F.54}$$

$$r_5=T_1^2T_2^2T_3^2 \tag{F.55}$$

$$r_6=T_1T_2T_3\rho_1+D\gamma_2. \tag{F.56}$$

The proof of the theorem is completed by choosing $\epsilon=\min\{\epsilon_1,\epsilon_2,\epsilon_3\}$.

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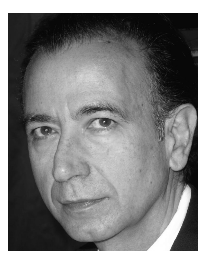
Nikolaos Bekiaris-Liberis received the Ph.D. degree from the University of California at San Diego, La Jolla, in 2013. From 2013 to 2014, he was a Post-Doctoral Researcher with the Mobile Sensing Laboratory, University of California at Berkeley, Berkeley. He is currently a Post-Doctoral Researcher with the Dynamic Systems and Simulation Laboratory, Technical University of Crete, Greece. He has co-authored the SIAM book *Nonlinear Control under Nonconstant Delays*. His interests are in delay systems, distributed parameter systems, nonlinear control, and their applications.

Dr. Bekiaris-Liberis was a finalist for the Student Best Paper Award at the 2010 ASME Dynamic Systems and Control Conference and at the 2013 IEEE Conference on Decision and Control. He received the Chancellor's Dissertation Medal in engineering from the University of California at San Diego in 2014 and the Best Paper Award at the 2015 International Conference on Mobile Ubiquitous Computing, Systems, Services, and Technologies. He was a recipient of the 2016 Marie Sklodowska-Curie Individual Fellowship Grant.

Claudio Roncoli received the bachelor's and M.Sc. degrees in computer science engineering and the Ph.D. degree in system monitoring and environmental risk management from the University of Genova, Italy, in 2006, 2009, and 2013, respectively.

He was a Research Assistant with the Department of Informatics, Bioengineering, Robotics, and Systems Engineering, University of Genova, from 2007 to 2013, and a Visiting Research Assistant with the Centre for Transport Studies, Imperial College London, U.K., in 2011 and 2012. From 2013 to 2016, he was a Post-Doctoral Research Associate with the Dynamic Systems and Simulation Laboratory, Technical University of Crete, Chania, Greece. Since 2016, he has been an Assistant Professor of transportation engineering with Aalto University, Finland.

His research interests include real-time traffic management, optimisation, and control of traffic and transportation systems, and methodologies to support smart mobility.



Markos Papageorgiou (F'99) received the Dipl.-Ing. and Dr.-Ing. degrees (Hons.) in electrical engineering from the Technical University of Munich, Germany, in 1976 and 1981, respectively. He was a Free Associate with Dorsch Consult, Munich, from 1982 to 1988, and the Institute National de Recherche sur les Transports et leur Scurit, Arcueil, France, from1986 to 1988. From 1988 to 1994, he was a Professor of automation with the Technical University of Munich. Since 1994, he has been a Professor with the Technical University of Crete, Chania, Greece. He was a Visiting Professor with the Politecnico di Milano, Italy, in 1982, the Ecole Nationale des Ponts et Chausses, Paris, from 1985 to 1987, and MIT, Cambridge, from 1997 to 2000, and a Visiting Scholar with the University of California at Berkeley, Berkeley, in 1993, 1997, 2001, and 2011, and other universities.

He has authored or edited over five books and 450 technical papers. His research interests include automatic control and optimisation theory and applications to traffic and transportation systems, water systems, and further areas. He was the Editor-in-Chief of *Transportation Research Part C* from 2005 to 2012. He is a fellow of IFAC in 2013. He received several distinctions and awards for his achievements. He was a recipient of the ERC Advanced Investigator Grant (2013–2018).